Written Assignment #6:
Linear Operators
Due 5:00pm Tuesday, October 7, 2003

You are encouraged to collaborate with your colleagues. For credit, however, your final write-up must be done individually. Show all your work and make your presentation comprehensible.

1. Determine whether or not the following operators are linear.
   (a) \( L_y = x^2 D^2 y - 2y; \)
   (b) \( L_y = y D^2 y + (Dy)^2; \)

2. (a) With \( L_y = x^2 D^2 y - 2y, \) write \( L_y = 0 \) as a differential equation. Verify that
   \[ y_1(x) = x^2 \text{ and } y_2(x) = \frac{1}{x} \]
   are solutions to this differential equation, for \( x > 0. \) If \( C_1 \) and \( C_2 \) are constants, is the linear combination \( C_1 y_1(x) + C_2 y_2(x) \) a solution to this differential equation, for \( x > 0? \)

   (b) With \( L_y = y D^2 y + (Dy)^2, \) write \( L_y = 0 \) as a differential equation. Verify that
   \[ y_1(x) = 1 \text{ and } y_2(x) = \sqrt{x} \]
   are solutions to this differential equation, for \( x > 0. \) If \( C_1 \) and \( C_2 \) are constants, is the linear combination \( C_1 y_1(x) + C_2 y_2(x) \) a solution to this differential equation, for \( x > 0? \)