Written Assignment #11:
Solving Differential Equations Using Laplace Transforms
Due 5:00pm Tuesday, November 11, 2003

You are encouraged to collaborate with your colleagues. For credit, however, your final write-up must be done individually. Show all your work and make your presentation comprehensible.

1. Use Laplace transforms to solve the following initial value problems.
(a) \(y'' + 10y' + 9y = f(t)\); \(y(0) = 1, y'(0) = 2\).
Express your answer using convolutions.
(b) \(y'' + 10y' + 9y = t^2e^{-9t}\); \(y(0) = 1, y'(0) = 2\).
You may use your result from part (a).

2. For this problem, suppose that the forcing function \(f\) is given by the following formula
\[f(t) = -20 + 40 \sum_{n=0}^{\infty} (-1)^n u(t - n\pi),\]
where \(u\) is the unit step function.
(a) Make a sketch for the graph of \(f\).
(b) Compute \(\mathcal{L}\{f(t)\}\).
(c) A mass-spring system has a mass of 1g, a spring constant of 20 g/sec\(^2\), and a damping constant of 4 g/sec. Suppose that the system is initially at rest at equilibrium, so \(x(0) = x'(0) = 0\), and that the mass is acted on by an external force given by \(f(t)\). Find the position function for the mass; i.e. find \(x(t)\).