and during the examination above instructions for this exam, and I will neither give nor receive any unauthorized aid.

Please:

- If it is determined that you have given or received any unauthorized aid, you will receive no credit for your exam.
- If you wish to speak with a proctor during the exam, then raise your hand and a proctor will come to you.
- If you wish to speak with a proctor during the exam, then raise your hand and a proctor will come to you.
- Write your solutions in an explicit form whenever possible.
- Study the questions carefully. If you encounter any difficulty, please seek assistance from a proctor.
- To receive full credit for a problem, you must show all your work that leads to the answer provided, and a sufficient amount of work that demonstrates your understanding of the problem.
- You may not use any other reference materials except for the course textbook and the provided exam sheet.
- You may not use a calculator during the exam.
- You may only use a calculator during the exam from 9:00am to 9:00pm. If you wish to receive credit for the exam, you must have a calculator.
- The examination period is from 7:00pm to 11:00pm.

Before you begin, make sure that your exam has not expired. If you do not do so, then you will not receive any credit for your exam.

There are 9 pages in this exam with 8 problems. Turn off all communication devices. If you do

Instructions

Exam 1

Kansas State University
Fall 2003

Elementary Differential Equations

Math 240
\[ y = c(x + 1)^3 \]

\( y = 0 \) is not a singular solution.

***Equation (use c = 0).***

\( y = 0 \) is represented in general.

\( y = 0 \) is a solution.

is \( \emptyset \) at \( 0 \).

Step 4: In step 3, we obtained by \( y \), which

\[ y = c(x + 1)^3 \]

Step 3: \( |y| = |m| \cdot |x + 1| + c \)

\[ x \cdot \frac{1 + x}{3} \int \frac{1}{1} y = y \cdot \frac{1}{1} \]

\[ \text{Step 1:} \quad -1 < x, \quad \frac{1 + x}{3} = \frac{xp}{\eta p} \]

1. Find all solutions to the following separable equation.
\[ \frac{x^2}{1 + x^2} = \gamma \]
\[ \frac{x^2}{1 + x^2} = \gamma \]
\[ 0 = x - x + \sqrt{x - 1}x \]

General Solution \( \gamma = x + \sqrt{x - 1}x \)

Step 6: \( \gamma = (1, \gamma) \)

Step 5: \( \gamma = (x, \gamma) \)

Step 4: \( \gamma = \frac{x \gamma + \sqrt{x - 1}x}{1 + \sqrt{x - 1}x} \)

Step 3: \( \gamma = \frac{x + \sqrt{x - 1}x}{1 + \sqrt{x - 1}x} \)

Step 2: \( \frac{x - 1}{x} = \frac{\gamma}{x} \)

Step 1: \( \frac{x - 1}{x} = \frac{\gamma}{x} \)

\[ \{ x, x - 1 \} = \left[ x, x - 1 \right] \frac{x}{\gamma} = \frac{x^2}{\gamma} \]

\[ \{ 1 + \sqrt{x - 1}x, \gamma \} \frac{x}{\gamma} = \frac{1 + x}{\gamma} \]

Solutions:

2. Test the following equation for exactness and then find all of the
\[
\begin{align*}
\frac{\partial}{\partial y} y &= \frac{\partial}{\partial x} \left(\frac{1}{\sqrt{x}} + 2e^\theta x^2\right) \\
&= 2e^\theta x + 4e^\theta x^3
\end{align*}
\]

\[
\Rightarrow \quad \int_{x_1}^{x_2} \frac{\partial}{\partial y} y \, dy = \int_{x_1}^{x_2} \left(2e^\theta x + 4e^\theta x^3\right) \, dx
\]

\[
\Rightarrow \quad \int_{x_1}^{x_2} \left(2e^\theta x + 4e^\theta x^3\right) \, dx = \left[ \frac{e^\theta x^2}{2} + e^\theta x^4 \right]_{x_1}^{x_2}
\]

\[
\Rightarrow \quad \int_{x_1}^{x_2} \left(2e^\theta x + 4e^\theta x^3\right) \, dx = \left(\frac{e^\theta x^2}{2} + e^\theta x^4 \right)_{x_1}^{x_2}
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\]

\[
\Rightarrow \quad \int_{x_1}^{x_2} \left(2e^\theta x + 4e^\theta x^3\right) \, dx = \left(\frac{e^\theta x^2}{2} + e^\theta x^4 \right)_{x_1}^{x_2}
\]

3. Solve the following initial value problem:
4. Solve the following initial value problem:

\[ y(2) = -1, \quad \frac{dy}{dx} = \frac{y}{x}, \quad x \neq 0 \]

**General Solution:**

\[ y = \frac{C}{x} \]

**Particular Solution:**

\[ y = \frac{C}{x} \]

\[ y(2) = -1 \]

\[ \frac{C}{2} = -1 \]

\[ C = -2 \]

\[ y = \frac{-2}{x} \]
No, it is not a streamline solution (a good choice).

\[ \left\{ \begin{array}{c}
 x = \frac{15}{e} - 3x \\
 y = \frac{15}{e} + (\frac{e}{2}x)^{2}
\end{array} \right. \text{ and } y = 0 \]

\[ y = \int \left[ \frac{15}{e} - \frac{1}{x} \right] dx + \left( \frac{e}{2}x \right)^2 = 0 \Rightarrow \left( \frac{e}{2}x \right)^2 + \frac{15}{e} x = \frac{15}{e} - \frac{1}{x} \]

Is it represented in the general solution?

Step 1: \( y = 0 \) is a solution. To find the general solution:

\[ \int \frac{dx}{x} = \int \left( \frac{15}{e} - \frac{1}{x} \right) dx + \left( \frac{e}{2}x \right)^2 \]

Step 2: Solve for \( \int \frac{dx}{x} \):

\[ \int \frac{dx}{x} = \frac{15}{e} x - \ln|x| + C \]

Step 3: \( C \) is a constant (arbitrary).

\[ \int \frac{dx}{x} = \frac{15}{e} x - \ln|x| + C \]

\[ 5x - 10y = 0 \Rightarrow \frac{xy}{yp} \]

5. Find all solutions to the following equation.
6. Suppose that \( p \) is the solution to the following initial value problem:

\[
\begin{align*}
\frac{dp}{dt} &= f(t, p) = 5, \\
p(t) &= g(t) = 0, \\
p(r) &= p = 3 \\
p(t) &= 3. \\
0 < p < 3, & \text{ if } 0 < p < 3, \\
1 < p < 3, & \text{ if } 1 < p < 3, \\
0 < p < 2, & \text{ if } 0 < p < 2.
\end{align*}
\]

(c) What value does \( p(t) \) approach as \( t \) decreases from 0? If \( p(0) \) decreases to \(-\infty\) or decreases to \(+\infty\), indicate so.

(d) What value does \( p(t) \) approach as \( t \) decreases from 0? If \( p(0) \) increases to \(+\infty\) or decreases to \(-\infty\), indicate so.

8. Based on the work in part a,

To \(+\infty\) it finite time \( t \),

and the solution increases.

So \( p(t) < 3 \).

\[ p(0) = 5 < 3 \]

\[ 0 < (p+2)(p+3) \]

If \( 0 < p < 3 \), then \( (p+2)(p+3) > 0 \).

If \( 1 < p < 3 \), then \( (p+2)(p+3) > 0 \).

If \( 0 < p < 2 \), then \( (p+2)(p+3) < 0 \).

Step 1: Check points \( p = 3 \) and \( p = -2 \).

\[ \frac{dp}{dt} = \frac{5p}{p^2} \]

The solution decreases to \(-\infty\) or decreases to \(+\infty\) for a finite \( t \), indicate so.

\( (t) \)
(a) Write an autonomous differential equation for this model: assume that the constant of proportionality is positive.

$$\frac{dP}{dt} = kP(N-P)$$

(b) Find the equilibrium points for your differential equation and determine their stability.

1. $P = 0$, since $kP(0-P) = 0$.
2. $P = N$, since $kP(N-P) = 0$.
3. $\frac{dP}{dt} < 0$ for $0 < P < N$.
4. $\frac{dP}{dt} > 0$ for $P > N$.

(c) According to this model, if at some time there is at least one person in the population that receives a piece of information, then how many people will eventually receive this information?

The solution to the population size $N$, the population size $P$, and the rate of change of the population size $\frac{dP}{dt}$.

According to the model, if at some time there is at least one person in the population that receives a piece of information, then how many people will eventually receive this information?
The model above, based on the model above, how many fish are harvested in a one-year period? Extra Credit (2 points): How many fish are harvested in a one-year period?

\[
\frac{1}{100} = \left( 1 - \frac{0.05 \cos(2\pi t)}{2.25} \right) = \frac{1}{100} \int_{0}^{0.5} \sin(\pi t) (2.25 + 0.05) dt
\]

These are about 2,232 fish which is

\[
P(1) \approx 2232.
\]

So, 2232 fish are harvested.

\[
\left[ \frac{2}{(0.05)^2 + (0.8)^2} \right] 80 = \int_{0}^{\infty} e^{-0.05t} dt
\]

\[
0.136 = \int_{0}^{\infty} e^{-0.05t} dt
\]

Now many fish are in the pond when \( t = 1 \)? Approximately the value of \( P(1) \). Approximately the size of \( y = 2.25 \) to approximate the value of \( y = 2.25 \).

Assume that \( P(0) = 80 \). Use the improved Euler's method with \( h = 0.5 \), \( \tau = 1 \), \( p = 1 \), \( \omega = 0.5 \).

\[
\tau = 0.5, \quad p = 0.5, \quad \omega = 0.5
\]

\[
P(t) \approx \left( 1 - \frac{0.05 \cos(2\pi t)}{2.5} \right) dt = \left( 1 - \frac{0.05 \cos(2\pi t)}{2.5} \right) dt
\]

\[
10000 \cdot 100 \sin(2\pi t) \left( 1 - \frac{0.05 \cos(2\pi t)}{2.5} \right) dt = \frac{1000}{d} dp
\]

The following differential equation models a pond's population of fish that grows logistically and is harvested periodically.

\[
\frac{dy}{dt} = \frac{10000}{d} \sin(2\pi t) \left( 1 - \frac{0.05 \cos(2\pi t)}{2.5} \right)
\]

where \( d \) is the size of the population in 100s of fish and \( t \) is in years.