Math 221 – Spring 2005
Practice Exam 3

75 minutes. Closed book. You are allowed a calculator and one 8.5” x 11” sheet of
handwritten notes. Not all problems on Exam 3 will have a matching problem on this
practice test, but this practice test should give you a sense of what the exam will be like
as you study. You will be provided with a table of integrals for the exam. You will need
to show all your work to receive full credit.

1. Evaluate \( \int_{-2}^{4} \frac{dx}{x^2 + 6x + 5} \). You need to give the exact answer, not a decimal
   approximation.

2. Suppose \( a_{n+2} = \frac{3}{2} a_{n+1} - \frac{1}{2} a_n \), with \( a_0 = 1 \) and \( a_1 = 1/2 \). Find \( \lim_{n \to \infty} a_n \). For partial credit
   you can just compute the first several terms and guess the limit. For full credit
   compute the first several terms, guess the formula for \( a_n \), show the formula satisfies
   the recurrence relation, and evaluate the limit using the formula.

3. Evaluate \( \sum_{n=0}^{\infty} \frac{2}{n^2 + 6n + 8} \).

4. Evaluate \( \frac{2}{3} + \frac{1}{6} + \frac{1}{24} + \frac{1}{96} + \ldots \).

5. Does \( \sum_{n=0}^{\infty} \frac{2n + 3}{n^2 + 4n + 1} \) converge conditionally, converge absolutely, or diverge?

6. What is the 2\(^{nd}\) degree Taylor polynomial approximation for \( x \sin(x) \) about \( x = \pi \)?

7. What is the radius of convergence of \( \sum_{n=0}^{\infty} \frac{n(x-1)^n}{3^n} \)?

8. What is an upper bound for the error in the approximation \( \cos(2x) \approx 1 - 2x^2 + \frac{2x^4}{3} \) at
   the point \( x = 2 \)? You must justify the upper bound from the error formula, not just
   evaluate the difference in the two values by trusting your calculator.

9. A child drops a super ball on a hard floor from a distance of 1 meter. Each time the
   ball bounces, it comes back up to 80% of its previous height. The first time the ball
   fails to bounce 10cm high, the child grabs the ball. Assuming the ball’s motion was
   straight up and down, how far did the ball travel?
10. Match the following sequences with their graphs.

a. \( a_n = \frac{5^n}{n!} \)

b. \( b_n = \frac{n^2 + 1}{n^3 + 1} \)

c. \( c_n = \sum_{k=0}^{n} \frac{2}{3^k} \)

d. \( d_n = \sum_{k=0}^{n} \frac{1}{k^2 + 3k + 2} \)