Math 221 – Exam 0
August 24, 2004

1. The tangent line to the graph \( y = 3x^2 - 4\sqrt{x} \) at (4,40) is
   a. \( y = 23x - 52 \)
   b. \( y = 63x - 212 \)
   c. \( y = 23x + 40 \)
   d. \( y = 63x + 40 \)
   e. \( y = -63x + 292 \)

Differentiate to get \( y' = 6x - 2x^{-1/2} \), so \( y'(4) = 23 \). The equation of the line through (4,40) with slope 23 is given in a. See section 2.1.

2. The derivative of \( f(x) = \sin(x)\cos(x) \) is
   a. \( f'(x) = -\sin(x)\cos(x) \)
   b. \( f'(x) = 1 \)
   c. \( f'(x) = \cos^2(x) - \sin^2(x) \)
   d. \( f'(x) = \frac{1}{2}\sin(2x) \)
   e. \( f'(x) = -\cos(\cos(x))\sin(x) \)

Use the product rule, remembering the derivative of \( \sin(x) \) is \( \cos(x) \) but the derivative of \( \cos(x) \) is \(-\sin(x)\), to get e. See section 2.3.

3. If \( f(x) = \frac{x^2 + 2}{3x - 1} \) then \( f''(2) \) is
   a. 1.2
   b. \( \frac{2}{25} \)
   c. 1.52
   d. \(-0.08 \)
   e. \( \frac{4}{3} \)

Use the quotient rule to get \( f'(x) = \frac{(3x-1)(2x)-(x^2+2)(3)}{(3x-1)^2} \) so \( f'(2) \) is b. See section 2.3.
4. The derivative of \( f(x) = \cos^3(2x) \) is
   a. \( f'(x) = -6\cos^2(2x)\sin(2x) \)
   b. \( f'(x) = 3\cos^2(2x) \)
   c. \( f'(x) = -3\cos^2(2x)\sin(2x) \)
   d. \( f'(x) = 6\cos^2(x) \)
   e. \( f'(x) = -3\sin^2(2x) \)

The chain rule gives \( f'(x) = 3\cos^2(2x)(-\sin(2x))(2) \) which simplifies to a. See section 2.4.

5. Evaluate \( \int 2x\sqrt{3x^2+1} \, dx \).
   a. \( \frac{2}{3} (3x^2+1)^{3/2} + C \)
   b. \( \frac{1}{6} (3x^2+1)^{-1/2} + C \)
   c. \( \frac{2}{3} x^2(x^3+x)^{3/2} + C \)
   d. \( x^2\sqrt{x^3+x} + C \)
   e. \( \frac{2}{9} (3x^2+1)^{3/2} + C \)

Make the substitution \( u = 3x^2 + 1 \), then \( du = 6xdx \). Since we already have a \( 2x \) in the integrand, we pick up a factor of \( \frac{1}{3} \) and the integral becomes \( \frac{1}{3} \int \sqrt{u} \, du = \frac{2}{9} u^{3/2} + C \).

Back substituting for \( u \) gives e. See section 4.5.

6. Evaluate \( \int_0^4 x^2 - \sqrt{x} \, dx \).
   a. 7.75
   b. 14
   c. \( \frac{58}{3} \)
   d. -14
   e. 16

\[ \int_0^4 x^2 - \sqrt{x} \, dx = \left[ \frac{x^3}{3} - \frac{2}{3} x^{3/2} \right]_0^4 = \left( \frac{4^3}{3} - \frac{2}{3} 4^{3/2} \right) - \left( 0^3 - \frac{2}{3} 0^{3/2} \right) \] which gives e. See section 4.3.
7. If \( x^2 - 2x \cos(y) + y^2 = 1 \), then \( y' (x) \) is
   a. \(-\frac{x}{y + \sin(y)}\)
   b. \(\frac{\cos(y) - x}{x \sin(y) + y}\)
   c. \(2x + 2x \sin(y) - 2 \cos(y) + 2y\)
   d. \(2x + 2x(\sin(\cos(x))) - 2 \cos(x)\)
   e. \(4x - 2 \cos(x) + 2x \sin(x)\)

Use implicit differentiation to obtain \(2x - (\cos(y) - 2x \sin(y))y' + 2yy' = 0\), then solve for \(y'\) to get b. See section 2.5.

8. The minimum value of \(x^3 - x + 1\) in the interval \([-1,2]\) is
   a. 0
   b. 1
   c. 7
   d. \(\frac{9 - 2\sqrt{3}}{9}\)
   e. \(\frac{9 + 2\sqrt{3}}{9}\)

Potential extreme values will be at critical points, where \(3x^2 - 1 = 0\), and at the endpoints.
In this case the minimum is d attained at the critical point \(x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}\). See section 3.1.

9. The maximum value of \(x^3 - x + 1\) in the interval \([-1,2]\) is
   a. 0
   b. 1
   c. 7
   d. \(\frac{9 - 2\sqrt{3}}{9}\)
   e. \(\frac{9 + 2\sqrt{3}}{9}\)

Potential extreme values will be at critical points, where \(3x^2 - 1 = 0\), and at the endpoints.
In this case the maximum is c attained at the endpoint \(x = 2\). See section 3.1.
10. Grain is pouring into a conical pile with radius equal to twice the height. If the pile is 4 feet high and rising at a rate of 1 foot per minute, how fast is the grain pouring? The formula for volume of a cone is \( V = \frac{1}{3} \pi r^2 h \).

   a. \( \frac{32}{3} \pi \) ft\(^3\) / min
   b. \( \frac{64}{3} \pi \) ft\(^3\) / min
   c. \( 64 \pi \) ft\(^3\) / min
   d. \( \frac{256}{3} \pi \) ft\(^3\) / min
   e. \( \frac{320}{3} \pi \) ft\(^3\) / min

Since \( r = 2h \), the formula becomes \( V = \frac{1}{3} \pi (2h)^2 h = \frac{4}{3} \pi h^3 \). Differentiate this to get \( V' = 4\pi h^2 h' \) and then plug in \( h = 4 \) and \( h' = 1 \) to get e. See section 2.2.

11. A 13 foot ladder is leaning against a wall. If the ladder is pulled away from a wall at the rate of 6 feet per second, how fast will the top of the ladder be sliding along the wall when the base is 5 feet from the wall?
   a. \(-12\) ft / sec
   b. \(-9.6\) ft / sec
   c. \(-2.5\) ft / sec
   d. \(\frac{-30}{13}\) ft / sec
   e. \(\frac{-5}{12}\) ft / sec

If \( x \) is the distance from the wall to the base of the ladder and \( y \) is the distance from the ground to the top of the ladder, we have \( x^2 + y^2 = 13^2 \) by the Pythagorean theorem. We differentiate both sides to get \( 2xx' + 2yy' = 0 \), then plug in \( x = 5, x' = 6, y = 12 \) (found from the Pythagorean theorem) and solve for \( y' \) to get e. See section 2.6.
12. The volume of the solid of revolution formed by rotating the region between the 
$x$–axis, the line $x = 2$, and the curve $y = x^2$ about the $x$–axis is 

a. $\frac{8}{3}$ units$^3$

b. $\frac{16}{3} \pi$ units$^3$

c. $\frac{32}{5} \pi$ units$^3$

d. $8 \pi$ units$^3$

e. $\frac{128}{5} \pi$ units$^3$

Use the method of disks to write $V = \int_0^2 \pi y^2 \,dx = \int_0^2 \pi x^4 \,dx$ which yields e. See section 6.2.