1. Solve for $a$ if \( \int_{1/2}^{a} \frac{dx}{x} = 3 \).

\[
\int_{1/2}^{a} \frac{dx}{x} = \ln \frac{a}{1/2} = \ln a - \ln \frac{1}{2} = \ln (\frac{a}{2}) \]

Set \( \frac{a}{2} = e^3 \),

\[
\frac{a}{2} = e^3
\]

\[
a = 2e^3
\]
2. A bottle of wine is taken out of a 50° wine cellar at 5:30 and allowed to 
warm in a 70° room. After 20 minutes the wine’s temperature is 58°. 
How warm will the wine be at 6:30?

\[ \text{Temperature} = 70 + Ce^{kt} \]

\[ T(0) = 50 \Rightarrow 70 + Ce^{k \cdot 0} \]

\[ 50 = 70 + C \]

\[ -20 = C \]

\[ 58 = T(20) = 70 - 20e^{k \cdot 20} \]

\[ -12 = -20e^{20k} \]

\[ e^{0.6} = e^{-20k} \]

\[ \ln(0.6) = 20k \]

\[ k = \frac{\ln(0.6)}{20} \]

at \( t = 60 \)

\[ T(60) = 70 - 20e^{\frac{\ln(0.6)}{20} \cdot 60} \]

\[ = 70 - 20e^{\ln(0.6)^3} \]

\[ = 70 - 20 \cdot (0.6^3) \]

\[ = 65.68 \approx 66° \]
3. Find the centroid of the trapezoid with corners at (3,1), (5,1), (3,5), and (7,5).

Subdivide the trapezoid into two simpler shapes,
(A) a rectangle with corners (3,1), (5,1), (3,5), (5,5)
(B) a triangle with corners (5,1), (5,5), (7,5)

Rectangle A has area 8 and centroid \( (4, 3) \) \( \left( \frac{(3,1)+(5,1)+(3,5)+5,5)}{4} \)\\
Triangle B has area 4 and centroid \( \left( \frac{17}{3}, \frac{11}{3} \right) \) \( \left( \frac{(5,1)+(5,5)+7,5)}{3} \)\\

So the trapezoid has centroid
\[
\bar{x} = \frac{8 \cdot 4 + 4 \cdot \frac{17}{3}}{8+4} = \frac{32 + \frac{68}{3}}{12} = \frac{164}{36} = \frac{41}{9}
\]
\[
\bar{y} = \frac{8 \cdot 3 + 4 \cdot \frac{17}{3}}{8+4} = \frac{24 + \frac{68}{3}}{12} = \frac{116}{36} = \frac{29}{9}
\]

\[ \left( \frac{41}{9}, \frac{29}{9} \right) \]
4. Evaluate \( \int x^2 e^{-2x} \, dx \).

Using tabular method of integration by parts:

\[
\begin{array}{c|c}
\text{u} & \text{dv} \\
\hline
x^2 & e^{-2x} \\
2x & -\frac{1}{2} e^{-2x} \\
2 & -\frac{1}{4} e^{-2x} \\
0 & -\frac{1}{8} e^{-2x} \\
\end{array}
\]

\[-(\frac{x^2}{2} + \frac{x}{2} + \frac{1}{4}) e^{-2x} + C\]