Analytic Geometry and Calculus I – Exam 1
Summer 2006

Show all work for full credit. You may use a basic function calculator, but not a graphing or scientific calculator. No notes or books are allowed.

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(10 pts) 1. Sketch the graph of the following function and use it to determine the values of $a$ for which $\lim_{x \to a} f(x)$ exists.

$$f(x) = \begin{cases} 
  x^2 + 1, & \text{if } x < -1 \\
  2x, & \text{if } -1 \leq x < 2 \\
  -x + 4, & \text{if } x \geq 2 
\end{cases}$$
2. Evaluate the following limits (if the limit does not exist, explain why).

(a) \( \lim_{x \to 1} \left( x^2 + \frac{3}{x + 2} \right) \)

(b) \( \lim_{x \to 3} \frac{2x^2 - 5x - 3}{x - 3} \)

(c) \( \lim_{h \to 0} \frac{(3 + h)^2 - 9}{h} \)

(d) \( \lim_{x \to 6} \frac{x^2 - 36}{\sqrt{x} - \sqrt{6}} \)

(e) \( \lim_{x \to 3} \frac{|x - 3|}{x - 3} \)

(f) \( \lim_{y \to \infty} \frac{2 - 3y^2}{5y^2 + 4y} \)
(10 pts) 3. By calculating an appropriate limit, find the slope of the tangent line to the graph of the function \( f(x) = x^2 \) at (3,9).

(10 pts) 4. Explain why the function is discontinuous at 1 and sketch the graph.

\[
f(x) = \begin{cases} 
x^2, & \text{if } x \leq 1 \\
2x, & \text{if } x > 1
\end{cases}
\]
(10 pts) 5.

(a) From the graph of $f$, state the numbers at which $f$ is discontinuous and explain why.

(b) From the graph of $f$, state the open intervals on which $f$ is continuous.
(10 pts) 6. The graph shows the position function of a car. Use the shape of the graph to explain your answers to the following questions.

(a) Was the car going faster at A or B?

(b) At which point(s) was the car slowing down?

(c) At which point(s) was the car speeding up?

(d) What happened around point C?
(10 pts) 7. Find the constant \( c \) that makes \( g \) continuous on \((-\infty, \infty)\).

\[
g(x) = \begin{cases} 
   cx + 3c, & \text{if } x < 2 \\
   5x, & \text{if } x \geq 2
\end{cases}
\]

(10 pts) 8. Let \( f(x) = x^2 + 5 \). Explain why there is a number \( c \) such that \( f(c) = 12 \).