CALCULUS 1
Final Exam
May 11, 2005

Below you will find 20 problems, each worth 10 points. Solve the problems in the space provided. When writing a solution to a problem, show all work. No books or notes are allowed.

**Problem 1.** Find the vertical asymptotes of the function \( f(x) = \frac{2x^2 + x - 1}{2x^2 - 5x + 2} \).

**Problem 2.** Find the volume of the solid obtained by rotating the region bounded by \( y = \sqrt{x}, \ y = x, \ x = 0, \) and \( x = 1, \) about the \( x \)-axis.
Problem 3. (a) Compute the first and second derivatives of the function \( f(x) = \frac{x^2 + 1}{x} \), then indicate the intervals where \( f(x) \) is increasing/decreasing, and concave upward/downward.

(b) Sketch (use the white space on the right) the graph of \( f(x) \). Below list the asymptotes (if any).

Problem 4. Given \( 2y + \cos y = (2x + 1)^3 \), compute \( \frac{dy}{dx} \), using implicit differentiation.

Problem 5. Find \( \lim_{x \to -\infty} \frac{2x + 1}{\sqrt{x^2 - x}} \).
Problem 6. Find the area of the region bounded by the graphs of the equations $y = x^2$ and $y = 2 - x$.

Problem 7. Find the indefinite integral: $\int \frac{x}{\sqrt{x + 2}} \, dx$.

Problem 8. Compute the definite integral: $\int_{-\pi}^{\pi} \frac{\sin x}{\sqrt{2 + \cos x}} \, dx$. 
Problem 9. Find the value of the constant $\alpha$, so that the function

$$f(x) = \begin{cases} \frac{\sin 3x}{x}, & \text{if } x > 0 \\ 2x + \alpha, & \text{if } x \leq 0 \end{cases}$$

is continuous on the entire real line.

Problem 10. Find the absolute minimum, and the absolute maximum of $f(x) = 2x - 3\sqrt{x^2}$, on the interval $[-1, 2]$.

Problem 11. Solve the differential equation $\frac{dy}{dt} = 4y$, $y(1) = 1$. 
Problem 12. A rectangle has one vertex at the origin, one vertex on the positive $x$-axis, one vertex on the positive $y$-axis, and one vertex on the curve $y = \frac{1}{x}$, $x > 0$. Find the smallest possible perimeter of such a rectangle.

Problem 13. Find the derivative of the function $f(x) = \ln \sqrt{\frac{x^2 + 1}{x^2 + 4}}$.

Problem 14. Find the indefinite integral $\int \frac{x^2 + x + 1}{x + 1} \, dx$. 
Problem 15. Find the volume of the solid obtained by rotating the region bounded by \( y = \sqrt{x} \), \( y = 0 \), \( x = 0 \), and \( x = 1 \), about the line \( x = 2 \).

Problem 16. Find the indefinite integral \( \int \frac{\sec^2 x}{1 + \tan x} \, dx \).

Problem 17. Use the first derivative to decide if the function \( f(x) = x^2 + \frac{16}{x} \), \( x > 0 \) has an inverse.
Problem 18. Given that the function $f(x) = x^3 + 3x^2 + 10x + 5$ has an inverse, compute $(f^{-1})'(5)$.

Problem 19. Compute the second derivative of $f(x) = (x^2 - 2x + 2)e^x$, then find the inflection points.

Problem 20. Find the indefinite integral $\int \frac{e^{3x}}{4 + e^{3x}} \, dx$. 