Homework # 6
Math 220, Fall 2014

1. Find the tangent line to the given function at the given point.
   (a) \( y = \sin x \) at \( x = \frac{\pi}{6} \).
   (b) \( y = \tan x \) at \( x = \frac{\pi}{4} \).

2. Find the derivative of the following functions:
   (a) \( f(x) = x^2 \sin x \).
   (b) \( g(t) = t^2 e^t + 7 \cos t \).
   (c) \( h(\theta) = \theta^2 (\sin \theta + \cos \theta) \).
   (d) \( y = \frac{1 + \tan x}{1 - \tan x} \).
   (e) \( z = \frac{e^x + x \sin x}{e^x - x \cos x} \).

3. Find \( y^{(79)} \) (i.e. the 79th derivative of \( y \)), for the following cases:
   (a) \( y = e^x \).
   (b) \( y = x^9 \).
   (c) \( y = \sin x \).
   (d) \( y = e^{-x} \).

   Comment: With the exception of 3(d) everything so far should be done without the chain rule. (3d can be done by the quotient rule, but it is far easier to do it with the chain rule.) Starting now, you should expect to be using the chain rule a lot!

4. Differentiate the following functions. Simplify where appropriate. (Simplification is very important because you will typically want to find out where your derivatives are zero, and that can be next to impossible if you cannot simplify your answer.)
   (a) \( y = e^{5x} \).
   (b) \( y = e^{6t^2} = \exp(6t^2) \).
(c) \( y = t^2 e^{2t} \).

(d) \( y = \cos(\pi t) \).

(e) \( y = 3 \sin(5x + \pi/6) \).

(f) \( y = t^5 \cos(6t + 7) \).

(g) \( y = \sqrt{t^4 + 800} \).

(h) \( y = (1 + x^2 \sin(5x))^7 \).

(i) \( y = \sin(\ln(2 + \cos(5 + e^{7x}))) \).

(j) \( y = \sin(e^{2x}) \cdot \cos(e^{4x}) \).

(k) \[
y = \frac{x + e^{2x}}{x - e^{2x}}.
\]

(l) \[
y = \left(\frac{1 - x^2 e^{-x}}{1 + x^2 e^{-x}}\right)^2.
\]

(m) \[
y = \sqrt{1 + x^2}.
\]

(n) \[
y = \frac{x^2 + 4}{\sqrt{x^4 + 1}}.
\]

(o) \[
y = \frac{e^{2x} \sqrt{x^2 + 4}}{x^2}.
\]

(p) \[
y = \sqrt{\frac{1 + xe^{-2x}}{1 - xe^{-2x}}}.
\]

(q) \( y = \cos(\tan(\sec(\ln(2 + t^4)))) \).

(r) \[
y = x^2 \cdot \sqrt{\frac{5 - x^2 e^{-4x}}{5 + x^2 e^{-4x}}}.
\]
5. Differentiate the following functions. You can refer to the formulas in Section 3.8.

(a) \( y = 3^x \sin x \).
(b) \( y = \log_3(3x) \).
(c) \( y = \log_3(8x) \).
(d) \( y = \arctan(1 + t^4) \).
(e) \( y = t \arcsin(\sqrt{1-t^2}) \).
(f) \( y = [1 + \arccos(xe^{-3x})]^4 \).
(g) \( y = (1 + t^2)\arccot(5t) \).

Comments: Different books have different conventions about the definitions of the inverse to the secant function and the inverse to the cosecant function, so I won’t test on those functions at all. In terms of memorization, I won’t have you memorize the derivatives of the following functions:

- Any and all hyperbolic functions and/or their inverses. (I won’t test on these functions.)
- The inverse trig functions. (Any formula that you need will be on a cheat sheet.)
- Logarithms with a base other than “e.” (Any formula that you need will be on a cheat sheet.)
- Exponential functions with a base other than “e.” (Any formula that you need will be on a cheat sheet.)