Area the Hard way

1. Solve the Applied Project on page 288.

2. In this problem you will use a limit to compute the area bounded by the curves $y = e^x$, $x = a$, $y = 0$ and $x = 0$. The concept - fill in more and more rectangles is fairly intuitive. The hard part is just the algebra. Here are some hints:

(a) Expand $(z - 1)(z^3 + z^2 + z + 1)$.

(b) Guess what $(z - 1)(z^{n-1} + z^{n-2} + \ldots + z^2 + z + 1)$ is equal to. Use this to get a simple formula for $z^{n-1} + z^{n-2} + \ldots + z^2 + z + 1$.

(c) Recognize $\lim_{h \to 0} \frac{e^h - 1}{h}$ as $f'(x)$ for some $f(x)$ and some $x$. Use this to compute the limit. Then compute $\lim_{h \to 0} \frac{h}{e^h - 1}$.

(d) Draw a sketch of the curves $y = e^x$, $x = 7$, $y = 0$ and $x = 0$. Now draw 5 evenly spaced rectangles completely contained in this region.

(e) Label the coordinates of the upper left corner of each rectangle.

(f) Write out the combined area of all of the rectangles, but do not simplify your answer.

(g) Based on the last part guess the combined area of $n$ evenly spaced rectangles in the region bounded by the curves $y = e^x$, $x = a$, $y = 0$ and $x = 0$. Call this area $A_n$.

(h) Factor out an expression that is common to each summand in the previous part.

(i) Use the formula that you guessed in part b) to simplify your expression for $A_n$.

(j) Notice that $\lim_{n \to \infty} \frac{a}{n} = 0$. Substitute $h = a/n$ into the expression $A = \lim_{n \to \infty} A_n$ to get a limit of the form $A = \lim_{h \to 0} \text{blah}$.

(k) Use part c) to compute this limit.

This Take-Home project will be due Wed, November 2 at 7:30 in the homework boxes. We will tell you some things to help you with the second part in lecture.
Area the easy way

In this homework we introduce the second main concept in calculus – the integral. Just as the derivative represents the slope of the tangent line, the integral represents the (signed) area under the curve. Computing the integral by definition is fairly intuitive, but the algebra, is so hard that the only time you will have to do it is when you solve the second take-home project. The only thing to be careful of here is that you don’t mix up the short-cuts for derivatives and integrals.

Homework 9: Due, Friday, November 4

sec 5.1 page 378: 1, 2, 3
sec 5.2 page 392: 33, 34
sec 5.3 page 402: 2, 3, 5, 6, 19, 23, 29, 31, 32
sec 5.4 page 411: 25, 26, 27, 28, 31, 33, 35, 38
sec 5.5 page 420: 4, 6, 9, 11, 18, 28, 31, 36, 41, 57, 65