Separation anxiety

In this homework, we cover separable differential equations. This topic is covered in chapter 9. Differential equations give very accurate models of physical phenomena. The equation \( y' = ky \) is the entire reason why the curriculum committee added logarithms and exponentials to math 220. Sec 9.3 number 34 requires the following algebraic equation.

\[
\frac{1}{(a - x)(b - x)} = \frac{1}{b - a} \left( \frac{1}{a - x} - \frac{1}{b - x} \right)
\]

We also start the computation of volumes (section 6.2).

Homework 11: Due, Wednesday, November 30

sec 9.3 page 607: 3, 5, 8, 12, 14, 16, 32, 34H, 37

sec 9.4 page 620: 3, 4, 13, 15, 22

sec 6.2 page 452: 2, 3, 7, 8, 27, 29, 49, 50, 55
Take-Home project 3: Due, Tuesday, November 29

The first problem in this take-home project is an accurate model of the time that a ball takes to travel up and down. The rest of the questions describe where the number e comes from. You know that \( \int x^n \, dx = \frac{1}{n+1} x^{n+1} + C \) for \( n \neq -1 \). You have also seen that \( \int x^{-1} \, dx = \ln x + C \). We want to explain why. (You can read a second argument in section 5.6). When you solve problems 2- below DO NOT USE THE FORMULA \( \int x^{-1} \, dx = \ln x + C \).

1. Solve page 610 up or down.
2. Graph \( y = x^{-1} \)
3. Notice that \( L(x) = \int_1^x t^{-1} \, dt \) makes geometric sense (for \( x > 0 \) which we will assume throughout this project) even though the formula \( \int x^n \, dx = \frac{1}{n+1} x^{n+1} + C \) does not apply.
4. Explain what \( L(1) \) must be from the geometric picture.
5. Explain why \( \int_1^a t^{-1} \, dt = \int_1^a t^{-1} \, dt + \int_a^b t^{-1} \, dt \) from the geometric picture (Draw a picture labeling the three integrals).
6. Express \( \int_1^a t^{-1} \, dt \) using \( L(x) \) and \( \int_1^a t^{-1} \, dt \) using \( L(x) \).
7. Make the substitution \( t = a\tau \) into the integral \( \int_{t=a}^{t=ab} t^{-1} \, dt \). (When \( t = a \) what does \( \tau \) equal? When \( t = ab \) what does \( \tau \) equal? What is \( dt \)?)
8. Using the previous problem express \( \int_a^b t^{-1} \, dt \) using \( L(x) \).
9. What is the formula that relates \( L(ab) \) to \( L(a) \) and \( L(b) \)?