Review 3 (§4.1 – §4.7 and §9.1 – §9.4 )  
Math 205, Fall, 2005

1. Critical points and inflection points
   • a. Critical points, local maxima and local minima, first and second derivative tests.
   • b. Inflection points, concavity.
   • c. Sketch a graph of a function that reflects the information about the signs of the first derivative and the second derivative of the function on different intervals.

2. Global maxima and minima for a function on a closed interval. Let $x_1, x_2, \ldots, x_k$ be the critical points of $f(x)$ in $[a, b]$.

   Glo. Max $= \max\{f(a), f(x_1), f(x_2), \ldots, f(x_k), f(b)\}$,
   Glo. Min $= \min\{f(a), f(x_1), f(x_2), \ldots, f(x_k), f(b)\}$

3. Optimization and other applications.
   • Condition for maximizing profit: $C'(q) = R'(q)$ or $\pi'(q) = 0$.
   • Average Cost function $A(q) = \frac{C(q)}{q}$ which is minimized only if $A(q) = C'(q)$. Graphic explanation of average cost with a cost function curve.
   • Elasticity of demand.
   • Logistic model $P(t) = \frac{L}{1 + Ce^{-kt}}$: Determine the parameters $L$, $k$ and $C$; Understand carrying capacity, point of diminishing return; Given $C$ and 3 of the 4 quantities $t$, $P(t)$, $L$, and $k$, find the remaining one.

4. Functions of two variables.
   • Estimate partial derivatives for functions given by table data or by contour diagrams.
   • For a function $f(x, y)$ defined algebraically, find the first partial derivatives $f_x(x, y)$, $f_y(x, y)$ and the second partial derivatives $f_{xx}(x, y)$, $f_{xy}(x, y)$ and $f_{yy}(x, y)$, and evaluate these derivatives at a given point $(a, b)$.
   • Linear approximations: $f(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$, or $\Delta f \approx f_x \Delta x + f_y \Delta y$ where “$\Delta$” means “Change in”. 