1. Global maxima and minima for a function on a closed interval. Let \( x_1, x_2, \cdots, x_k \) be the critical points of \( f(x) \) in \([a, b]\).

Glo. Max = \( \max\{f(a), f(x_1), f(x_2), \cdots, f(x_k), f(b)\} \),

Glo. Min = \( \min\{f(a), f(x_1), f(x_2), \cdots, f(x_k), f(b)\} \),

2. Optimization and other applications.

- Condition for maximizing profit: \( C'(q) = R'(q) \) or \( \pi'(q) = 0 \).
- Average Cost function \( A(q) = \frac{C(q)}{q} \) which is minimized only if \( A(q) = C'(q) \). Graphic explanation of average cost with a cost function curve.
- Elasticity of demand.
- Logistic model \( P(t) = \frac{L}{1 + Ce^{-kt}} \): Determine the parameters \( L, k \) and \( C \); understand carrying capacity, point of diminishing return; Given \( C \) and 3 of the 4 quantities \( t, P(t), L, \) and \( k \), find the remaining one.

3. Functions of two variables.

- Estimate partial derivatives for functions given by table data or by contour diagrams.
- For a function \( f(x, y) \) defined algebraically, find the first partial derivatives \( f_x(x, y) \), \( f_y(x, y) \) and the second partial derivatives \( f_{xx}(x, y) \), \( f_{xy}(x, y) \) and \( f_{yy}(x, y) \), and evaluate these derivatives at a given point \((a, b)\).
- Linear approximations: \( f(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b) \), or \( \Delta f \approx f_x \Delta x + f_y \Delta y \) where “\( \Delta \)” means “Change in”.
- Find critical points of \( f(x, y) \) by solving

\[
\begin{cases}
  f_x(x, y) = 0 \\
  f_y(x, y) = 0;
\end{cases}
\]

Extrema of \( f(x, y) \) and related optimization problems. D-test using

\[
D(x, y) = f_{xx}f_{yy} - (f_{xy})^2.
\]