1. Assume $1000 is deposited on 1/1/07 to an account with a nominal rate of 8% interest compounded quarterly. Write down an expression for how much it will be on 1/1/27. How much would it be if the interest was compounded continuously?

$$1000 \left(1 + \frac{0.08}{4}\right)^{20} = 1000(1.02)^{20}.$$ \[= 1000 \cdot 1.6.\]

2. Assume that you want to have $10,000, ten years from now. How much would you have to invest immediately (one lump sum) if you could get 12% nominal interest compounded monthly? What if it was only compounded quarterly? What if it is compounded continuously?

$$10,000 = P (1.01)^{120}, \text{ so } P = 10,000 (1.01)^{-120}.$$ \[= 10,000 \cdot 1.\]

$$10,000 = P (1.03)^{40}, \text{ so } P = 10,000 (1.03)^{-40}.$$ \[= 10,000 \cdot 1.\]

$$10,000 = P e^{1.2}, \text{ so } P = 10,000 e^{-1.2}.$$ \[= 10,000 \cdot 1.\]

3. Work out the effective annual rate of interest for each of the rates in the two preceding problems.

$$(1.02)^4 - 1, \quad e^{0.08} - 1.$$ \[= (1.01)^{12} - 1, \quad (1.03)^4 - 1, \quad e^{1.2} - 1.$$ \[= 1.\]

4. Now suppose that you are saving up for retirement by depositing $100 each month starting 1/1/07 into an investment which will return 8% interest compounded semiannually. If these deposits continue until 1/1/57, when you retire, then write down an expression for how much you will have then. Be sure to simplify this expression. What if the interest is compounded continuously?
Compounding Semiannually:

Total = $100 (1.04)^{(2)(50)} + $100 (1.04)^{(2)(\frac{49}{12})} + \cdots
+ $100 (1.04)^{(2)(\frac{2}{12})} + $100 (1.04)^{(2)(\frac{1}{12})} + $100

= $100 \left[(1.04)^{100} + (1.04)^{99\frac{\frac{2}{2}}{2}} + \cdots + (1.04)^{\frac{2}{2}} + (1.04)^{\frac{1}{2}} + 1\right]

= $100 \left[(1.04)^{\left(\frac{1}{2}\right)}(6)(100) + (1.04)^{\left(\frac{1}{2}\right)}(6)(99\frac{\frac{2}{2}}{2}) + \cdots + (1.04)^{\left(\frac{1}{2}\right)}(6)(\frac{1}{2}) + 1\right]

= $100 \left[(1.04)^{\left(\frac{1}{2}\right)}\right]^{600} + (1.04)^{\left(\frac{1}{2}\right)} \left(\frac{599}{12}\right) + \cdots + (1.04)^{\left(\frac{1}{2}\right)} \left(\frac{1}{12}\right) + 1\right]

= $100 \left[\left(\frac{(1.04)^{\left(\frac{1}{2}\right)}}{12} - 1\right) \right]

Compounding Continuously:

Total = $100 e^{(0.08)(50)} + $100 e^{(0.08)(\frac{49}{12})} + \cdots
+ $100 e^{(0.08)(\frac{2}{12})} + $100 e^{(0.08)(\frac{1}{12})} + $100

= $100 \left[(e^{0.08})^{\left(\frac{1}{12}\right)}(12)(50) + (e^{0.08})^{\left(\frac{1}{12}\right)}(12)(99\frac{\frac{2}{2}}{2}) + \cdots + (e^{0.08})^{\left(\frac{1}{12}\right)}(12)(\frac{1}{12}) + 1\right]

= $100 \left[(e^{0.08})^{\left(\frac{1}{12}\right)}\right]^{600} + (e^{0.08})^{\left(\frac{1}{12}\right)} \left(\frac{599}{12}\right) + \cdots + (e^{0.08})^{\left(\frac{1}{12}\right)} \left(\frac{1}{12}\right) + 1\right]

= $100 \left[\left(\frac{(e^{0.08})^{\left(\frac{1}{12}\right)}}{12} - 1\right) \right]

5. Now redo the previous problem, but assume that your last deposit is
on 1/1/47. What will you have ten years later on 1/1/57 when you retire? Simplify your expression!

Compounding Semiannually:

\[
\text{Total} = 100 \cdot (1.04)^{(2)(50)} + 100 \cdot (1.04)^{(2)(49.5)} + \ldots \\
+ 100 \cdot (1.04)^{(2)(10)} + 100 \cdot (1.04)^{(2)(10)} \\
= 100 \left[ (1.04)^{100} + (1.04)^{99.5} + \ldots + (1.04)^{10} + (1.04)^{10} \right] \\
= 100 \left[ (1.04)^{80} + (1.04)^{79.5} + \ldots + (1.04)^{5} + 1 \right] \\
= 100 \left[ (1.04)^{480} + (1.04)^{479} + \ldots + (1.04)^{481} - 1 \right] \\
= 100 \left[ (1.04)^{481} - 1 \right] \\
\]
Compounding Continuously:

\[
\text{Total} = 100 \, e^{0.08(50)} + 100 \, e^{0.08(49.12)} + \ldots \\
+ 100 \, e^{0.08(10.12)} + 100 \, e^{0.08(10)} \\
= 100 \left[ e^{0.08(10)} \left( e^{0.08(12)(40)} + e^{0.08(12)(39.12)} + \ldots \right) \\
+ e^{0.08(12)(39.12)} + e^{0.08(12)(40)} + 1 \right] \\
= 100 \left[ e^{0.08(10)} \left( \left( e^{0.08(12)(40)} \right)^{480} + \left( e^{0.08(479)} \right)^{479} + \ldots \\
+ \left( e^{0.08(479)} \right)^2 + \left( e^{0.08(479)} \right)^1 + 1 \right] \\
= 100 \left[ e^{0.08(10)} \left( \frac{\left( e^{0.08(481)} \right)^{481} - 1}{\left( e^{0.08(479)} \right)^{481} - 1} \right) \right]
\]

6. Here are data for tests in three different sections.

Section 1: 80, 79, 78, 77, 76, 75, 40.

Section 2: 79, 78, 77, 76, 75, 74, 73.

Section 3: 99, 98, 97, 80, 15, 14, 13.

Rank the sections in terms of mean, median, and standard deviation. (You don’t need to compute these in order to rank them!)

\[
\text{Median(2)} = 76 < \text{Median(1)} = 77 < \text{Median(3)} = 80
\]

\[
\text{Mean(2)} = 76. \text{ I can’t compute the mean of Section 1 without my calculator, but because of the person who scored 40 the mean will certainly drop below the mean from section 2. It is not hard to estimate that } 68 < \text{Mean(1)} < 75. \text{ In fact, by observing that } 80 + 40 = 60 + 60 \text{ and that } 79 + 78 + 77 + 76 + 75 = 77 \times 5, \text{ I see that the mean would be the same as if the scores were 60, 60, 77, 77, 77, 77, 77, so looking at that I can be pretty sure } 71 < \text{Mean(1)} < 74. \text{ For the third section, I observe }
\]
that $99 + 13 = 98 + 14 = 97 + 15 = 56 + 56$, so I will get the same mean as if the scores were $80, 56, 56, 56, 56, 56, 56$, so $\text{Mean}(3) < 65$. Thus,

$$\text{Mean}(3) < \text{Mean}(1) < \text{Mean}(2).$$

The standard deviation measures how close to the mean the typical person’s score is. So, one can simply look to see:

$$\text{StdDev}(2) < \text{StdDev}(1) < \text{StdDev}(3).$$

7. Here are data for student’s homework averages and their test averages for three different sections.

Sec 1: (40, 80), (39, 79), (39, 78), (38, 77), (38, 76), (38, 75), (20, 40).

Sec 2: (37, 31), (42, 47), (58, 51), (62, 69), (78, 71), (83, 89), (99, 93).

Sec 3: (55, 99), (53, 98), (54, 97), (54, 80), (56, 15), (55, 14), (56, 13).

For each of the first two sections and to the nearest whole number, what will the slope of the least squares regression line be? Rank the sections in terms of the correlation between the homework average and the test average.

For slopes of the least squares regression lines for Sections 1 and 2, I get 2 and 1 respectively. These two are easy to “eyeball,” as the test scores for Section 1 are within 1 point of double the HW average for each student, and for the second section the test scores are always within 10 of the HW scores all the way from the thirties to the nineties, and the data jumps back and forth across the line $y = x$. The correlation for Section 1 will be super close to 1 as one can predict very well the exact score for the tests knowing the student’s HW average. The correlation for Section 2 will be high, but certainly less than the correlation for Section 1. Basically, for the second section you can predict fairly well how a student will do on their test from their HW average, but certainly not to within 1 point. When I made the data for Section 3, I was trying to make it so that HW would have no predictive value for the test scores (i.e. correlation close to zero), but I think that it has even become a negative correlation as the two best HW averages belong to two of the three worst tests. In any case, either way we have:

$$\text{Correlation}(3) < \text{Correlation}(2) < \text{Correlation}(1).$$
8. True or False: If Data Set A has a correlation of .9, and Data Set B has a correlation of .3, then the slope of the least squares regression line for Data Set A will be larger than the slope of the least squares regression line for Data Set B.

False. You can’t tell. Correlation doesn’t determine the slope of the least squares regression line. It does determine how close most of the points are to that line. The only connection between the value of the correlation and the slope of the least squares regression line is that they will have the same sign.

9. Find a 95% confidence interval for the percentage of voters that prefer Jane to John for an election to the senate assuming that 1000 voters are found at random, and 57% of them favor Jane. From this data, can you determine for sure who will win the election?

\[ 0.57 - 2 \sqrt{ \frac{(0.57)(0.43)}{1000} } \leq p \leq 0.57 + 2 \sqrt{ \frac{(0.57)(0.43)}{1000} } \]

No, you can’t determine who will win for sure, but Jane is a good guess if your sample was truly random.