PLANE TRIGONOMETRY

Exam I
September 13, 2007

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Below you will find 10 problems, each worth 10 points. Solve the problems in the space provided. When writing a solution to a problem, show all work. No books or notes are allowed. Sign and submit your formula sheet with the exam.

Problem 1. Convert the units as indicated:
(a) $12.35^\circ$ to degrees, minutes and seconds.

$$12.35^\circ = 12^\circ + 0.35^\circ = 12^\circ + 0.35 \cdot 60^\prime =$$

$$= 12^\circ + 21^\prime = \boxed{12^\circ 21^\prime}$$

(b) $\frac{7\pi}{72}$ (radians) to degrees, minutes and seconds.

$$\frac{7\pi}{72} \text{ (rad)} = \frac{7\pi}{72} \cdot \frac{180^\circ}{\pi} = \left(\frac{35}{2}\right)^\circ = 17^\circ + \left(\frac{1}{2}\right)^\circ =$$

$$= 17^\circ + \frac{1}{2} \cdot 60^\prime = \boxed{17^\circ 30^\prime}$$
Problem 2. Find two positive coterminal angles and one negative coterminal angle for each of the following angles:

(a) \(-230^\circ\) (use degrees);
\[-230^\circ + 360^\circ = \boxed{130^\circ}\]
\[-230^\circ - 360^\circ = \boxed{-590^\circ}\]

(b) \(\frac{9\pi}{11}\) (use radians).
\[\frac{9\pi}{11} + 2\pi = \frac{9\pi}{11} + \frac{22\pi}{11} = \boxed{\frac{31\pi}{11}}\]
\[\frac{9\pi}{11} - 2\pi = \frac{9\pi}{11} - \frac{22\pi}{11} = -\frac{13\pi}{11}\]

Problem 3. An angle \(\theta\), in standard position, is located in the third quadrant and has \(\cot \theta = \frac{20}{21}\). Find the exact values of \(\sin \theta\) and \(\cos \theta\).

\[\csc \theta = \pm \sqrt{1 + \cot^2 \theta} = \pm \sqrt{1 + \left(\frac{20}{21}\right)^2} = \pm \sqrt{1 + \frac{400}{441}} = \pm \sqrt{\frac{841}{441}} = \pm \frac{29}{21}\]

\(\sin \theta \) is in Q3, \(\csc \theta = -\frac{29}{21}\) and \(\theta = 29^\circ\).
\[\sin \theta = \frac{1}{\csc \theta} = \boxed{-\frac{21}{29}}\]
\[\cos \theta = \cot \theta \cdot \sin \theta = \frac{-20}{21} \cdot \left(-\frac{21}{29}\right) = \boxed{-\frac{20}{29}}\]

Problem 4. Find the exact values of \(\sin \left(-\frac{5\pi}{4}\right)\) and \(\cos \left(-\frac{5\pi}{4}\right)\).

\[\sin \left(-\frac{5\pi}{4}\right) = -\sin \frac{5\pi}{4} = -\sin \left(\pi + \frac{\pi}{4}\right) = -(-\sin \frac{\pi}{4}) = \frac{\sqrt{2}}{2}\]
\[\cos \left(-\frac{5\pi}{4}\right) = \cos \frac{5\pi}{4} = \cos \left(\pi + \frac{\pi}{4}\right) = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}\]

\(\sin(-t) = -\sin t\) \hspace{1cm} \(\cos(-t) = \cos t\)
\[\sin(t + \pi) = -\sin t\] \hspace{1cm} \[\cos(t + \pi) = -\cos t\]

Formulas used \(\uparrow\)
**Problem 5.** Find the length of the arc that subtends the angle $100^\circ$ on a circle of diameter 18 in.

\[
L = r\theta \quad \text{in radian:} \quad \theta = 100^\circ = 100 \cdot \frac{\pi}{180} = \frac{5\pi}{9}
\]

\[
\theta = \frac{\text{diam}}{2} = \frac{18}{2} = 9 \quad \text{SO}
\]

\[
L = 9 \cdot \frac{5\pi}{9} = \frac{5\pi}{1}
\]

**Problem 6.** Prove the identity: $\sin^2 t (\csc^2 t - 1) = \cos^2 t$.

\[
\text{LHS = } \sin^2 t \csc^2 t - \sin^2 t = \sin^2 t \cdot \frac{1}{\sin^2 t - \sin^2 t} = 1 - \sin^2 t = \cos^2 t
\]

\[
\text{Pythagorean identity: } \sin^2 t + \cos^2 t = 1
\]

**Problem 7.** Find the exact values of $t$, in the interval $[-\pi, 5\pi]$, which satisfy the equation

\[
\sin t = -\frac{\sqrt{2}}{2}
\]

First, we know that the only angle in Q1 with $\sin \theta = \frac{1}{2}$ is $\frac{\pi}{3}$. (Note that we solve the absolute equation here!)

Second, we know the solutions of $\sin t = -\frac{\sqrt{2}}{2}$

are in the list: $\frac{\pi}{4}, \frac{7\pi}{4}$, etc. (Note that we solve the absolute equation here!)

In our case (since $\sin$ is negative in Q3, Q4) it follows that the basic solutions are $\frac{11\pi}{4}, \frac{15\pi}{4}$.

Using the graph, the solutions are:

\[
\begin{align*}
\frac{5\pi}{4}, & \quad \frac{7\pi}{4}, \quad \frac{13\pi}{4} & \quad \frac{15\pi}{4} \\
\frac{11\pi}{4}, & \quad \frac{15\pi}{4}, \quad \frac{19\pi}{4} & \quad \frac{23\pi}{4}
\end{align*}
\]
Problem 8. Find the equation of the line that passes through the points $A(-2, 2)$ and $B(3, -8)$.

\[ \text{Slope } m = \frac{y_2-y_1}{x_2-x_1} = \frac{(-8)-2}{(3)-(-2)} = \frac{x_1}{-2} \frac{y_2}{2} \frac{x}{-8} y_2 \]

\[ m = \frac{-10}{5} = -2 \]

Equation: \[ y - y_1 = m(x-x_1) \]

\[ y - 2 = -2(x-(-2)) \]
\[ y - 2 = -2x - 4 \]

**Problem 9.** Find the equation of the line that passes through the point $P(2, -1)$ and is parallel to the line $3x - 4y = 7$.

*Given line can be re-written as* \[ 4y = 3x - 7 \text{, or } y = \frac{3}{4}x - \frac{7}{4} \text{. So}

*given line has slope* \( m_0 = \frac{3}{4} \). The unknown line (parallel to given one) has slope \( m = m_0 = \frac{3}{4} \).

*The unknown line has equation* \[ y - y_0 = m(x-x_0) \]

\[ y - (-1) = \frac{3}{4} (x - 2) \]
\[ y + 1 = \frac{3}{4}x - \frac{3}{2} \]

\[ y = \frac{3}{4}x - \frac{5}{2} \]

**Problem 10.** Find the equation of the line that passes through the point $Q(-1, 2)$ and is perpendicular to the line $x - 4y = 12$.

*Given line can be re-written as* \[ 4y = x - 12 \]
\[ y = \frac{1}{4}x - 3 \text{. So given line has slope* } m_0 = \frac{1}{4} \text{.}

The unknown line (perpendicular to given one) has slope \( m = -\frac{1}{m_0} = -\frac{1}{\frac{1}{4}} = -4 \).

The unknown line has equation \[ y - y_0 = m(x-x_0) \]

\[ y - 2 = 4(x-(-1)) \]
\[ y - 2 = 4x + 4 \]

\[ y = 4x + 6 \]