Below you will find 10 problems, each worth 10 points. Solve the problems in the space provided. When writing a solution to a problem, show all work. No books or notes are allowed. Sign and submit your formula sheet with the exam.

**Problem 1.** A rocket, flying on a straight vertical trajectory, is observed from a point positioned 1000 meters away from the launch site. During the final minute of flight, the angle of elevation of the rocket (measured from the observation point) changed from 64° to 78°. How long was the flight path of the rocket during this period? Approximate your answer to the nearest tenth of a meter.

We need $x = b - a$

where

$$\frac{a}{1000} = \tan 64^\circ$$

$$a = 1000 \tan 64^\circ$$

$$\frac{b}{1000} = \tan 78^\circ$$

$$b = 1000 \tan 78^\circ$$

So: $x = 1000 \tan 78^\circ - 1000 \tan 64^\circ \approx 2654.3 \text{ m}$
Problem 2. The graph of the equation \( y = a \sin(bx + c) \), with \( a > 0 \) and \( b > 0 \), is shown in the figure below.

(a) Find the amplitude and the period.

Amplitude = \[ 5 \]
Period = length of fundamental interval = \[ 8 \]

(b) Find the coefficients \( a \), \( b \) and \( c \).

\[ a = \text{Amp} = 5 \]
\[ b = \frac{2 \pi}{\text{Period}} = \frac{2 \pi}{8} = \frac{\pi}{4} \]
\[ c = -b \cdot \text{Start} = -\frac{\pi}{4} \cdot (-1) = \frac{\pi}{4} \]

Problem 3. Find the amplitude and the period for the curve \( y = 4 \sin \left( 2x + \frac{\pi}{3} \right) \).

Amplitude = \[ 4 \]
Period = \[ \frac{2 \pi}{|b|} = \frac{2 \pi}{2} = \pi \]

Problem 4. Verify the identity: \( (\csc \theta - \cot \theta)(1 + \sec \theta) = \tan \theta \).

\[
\text{LHS} = \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right) \left( 1 + \frac{1}{\cos \theta} \right) =
\]
\[
= \left( \frac{1 - \cos \theta}{\sin \theta} \right) \left( \frac{1 + \cos \theta}{\cos \theta} \right) =
\]
\[
= \frac{(1 - \cos \theta)(1 + \cos \theta)}{\sin \theta \cos \theta} = \frac{1 - \cos^2 \theta}{\sin \theta \cos \theta} =
\]
\[
= \frac{\sin^2 \theta}{\sin \theta \cos \theta} = \frac{\sin \theta}{\cos \theta} = \text{RHS}
\]
Problem 5. Find all solutions of the equation: \( \sin \left( \frac{4x + \pi}{3} \right) = \frac{1}{2} \). Use exact values.

\[
\sin \left( \frac{4x + \pi}{3} \right) = \frac{1}{2}
\]

Basic sol: \( \frac{\pi}{6} \), \( \pi - \frac{\pi}{6} = \frac{5\pi}{6} \), \( \pi + \frac{\pi}{6} = \frac{7\pi}{6} \), \( 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \)

\[
4x + \frac{\pi}{3} = \frac{\pi}{6} + 2n\pi
\]

\[
x = \frac{1}{4} \left[ \frac{\pi}{6} + 2n\pi \right]
\]

\[
x = \frac{1}{4} \left[ \frac{\pi}{2} + 2n\pi \right]
\]

Problem 6. Find the exact values of \( \sin \left( \frac{23\pi}{3} \right) \) and \( \cos \left( \frac{23\pi}{3} \right) \).

\[
\frac{23\pi}{3} = -\frac{\pi}{3} + 8\pi \quad \text{so}
\]

\[
\sin \left( \frac{23\pi}{3} \right) = -\sin \left( \frac{\pi}{3} \right) = -\frac{\sqrt{3}}{2}
\]

\[
\cos \left( \frac{23\pi}{3} \right) = \cos \left( -\frac{\pi}{3} \right) = \frac{1}{2}
\]

Problem 7. Find the solutions of the equation \( 4 \cos^2 t - 4 \cos t - 3 = 0 \), that are in the interval \([0, 2\pi]\). Use exact values

\[
\cos t = u \rightarrow 4u^2 - 4u - 3 = 0
\]

\[
u = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-3)}}{2(4)} = \frac{4 \pm \sqrt{64}}{8} = \frac{4 \pm 8}{8}
\]

\[
u_1 = \frac{12}{8} = \frac{3}{2} \quad \text{and} \quad \nu_2 = -\frac{4}{8} = -\frac{1}{2}
\]

\[
\cos t = \frac{3}{2}
\]

\[
\cos t = -\frac{1}{2}
\]

No sol. \( t = \text{outside} \left( \pi - \frac{\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3} \right) \)}

\[
t = \frac{2\pi}{3}
\]

\[
t = \frac{4\pi}{3}
\]
Problem 8. Verify the identity: \( \cos x + \sin x \tan x = \sec x \).

\[
\text{LHS} = \cos x + \sin x \tan x = \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \text{RHS}.
\]

Problem 9. Given the triangle \( \triangle ABC \), with \( \angle C = 90^\circ \), \( a = 3 \text{ cm} \), and \( b = 2 \text{ cm} \), find the remaining elements of the triangle: the side \( c \), and the angles \( \hat{A} \) and \( \hat{B} \). (When computing the angles, express them in degrees, rounded to the nearest tenth.)

\[
c = \sqrt{a^2 + b^2} = \sqrt{3^2 + 2^2} = \sqrt{13} \approx 3.61
\]

\[
\tan \hat{A} = \frac{a}{b} = \frac{3}{2} \Rightarrow \hat{A} = \tan^{-1} \left( \frac{3}{2} \right) \approx 56.3^\circ
\]

\[
\hat{B} = 90^\circ - \hat{A} \approx 33.7^\circ
\]

Problem 10. Find the solutions of the equation \( \tan^2 x - 2 \tan x - 4 = 0 \), that are in the interval \( \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \). Round to nearest tenth of a radian.

\[
\tan x = w \Rightarrow w^2 - 2w - 4 = 0
\]

\[
w = \frac{2 \pm \sqrt{2^2 - 4(-4)}}{2} = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}
\]

\[
x_1 = \tan^{-1} (1 + \sqrt{5}) \approx 1.3
\]

\[
x_2 = \tan^{-1} (1 - \sqrt{5}) \approx -0.9
\]