Below you will find 10 problems, each worth 10 points. Solve the problems in the space provided. When writing a solution to a problem, **show all work**. No books or notes are allowed. **Sign and submit your formula sheet with the exam.**

**Problem 1.** Convert the units as indicated:

(a) $12.44^\circ$ to degrees, minutes and seconds.

\[
12.44^\circ = 12^\circ + 0.44^\circ = 12^\circ + 0.44 \times 60^\prime = 12^\circ + 26.4^\prime = 12^\circ + 26^\prime + 0.4^\prime = 12^\circ + 26^\prime + 0.4 \times 60^\prime = 12^\circ 26^\prime 24^\prime.
\]

(b) $\frac{5\pi}{72}$ (radians) to degrees, minutes and seconds.

\[
\frac{5\pi}{72} = \frac{5\pi}{72} \times \frac{180^\circ}{\pi} = \frac{900^\circ}{72} = \frac{25^\circ}{2} = 12^\circ + \frac{1^\circ}{2} = 12^\circ 30^\prime.
\]
Problem 2. Find two positive coterminal angles and one negative coterminal angle for each of the following angles:

(a) $110^\circ$ (use degrees);

First positive angle could be: $110^\circ + 360^\circ = 470^\circ$. Second positive angle could be: $110^\circ + 2 \times 360^\circ = 730^\circ$. The negative angle could be: $110^\circ - 360^\circ = -250^\circ$.

(b) $-\frac{7\pi}{6}$ (use radians).

First positive angle could be: $-\frac{7\pi}{6} + 2\pi = -\frac{7\pi}{6} + \frac{12\pi}{6} = \frac{5\pi}{6}$. Second positive angle could be: $-\frac{7\pi}{6} + 4\pi = -\frac{7\pi}{6} + \frac{24\pi}{6} = \frac{17\pi}{6}$. The negative angle could be: $-\frac{7\pi}{6} - 2\pi = -\frac{7\pi}{6} - \frac{12\pi}{6} = -\frac{19\pi}{6}$.

Problem 3. An angle $\theta$, in standard position, is located in the third quadrant and has $\tan \theta = \frac{5}{13}$. Find $\sin \theta$ and $\cos \theta$. (HINT. Use the given information to find the signs for $\sin \theta$ and $\cos \theta$ first. Then use the fundamental identities.)

We have $\sec \theta = \pm \sqrt{1 + \tan^2 \theta} = \pm \sqrt{1 + \left(\frac{5}{13}\right)^2} = \pm \sqrt{1 + \frac{25}{169}} = \pm \sqrt{\frac{194}{169}} = \pm \frac{\sqrt{194}}{13}$.

Since $\theta$ is in quadrant III, we actually have $\sec \theta = -\frac{\sqrt{194}}{13}$, so we get $\cos \theta = \frac{1}{\sec \theta} = -\frac{13}{\sqrt{194}}$. This gives

$$\sin \theta = \cos \theta \cdot \tan \theta = -\frac{13}{\sqrt{194}} \cdot \frac{5}{13} = -\frac{5}{\sqrt{194}}.$$ 

Problem 4. Find the exact values of $\sin \left(-\frac{5\pi}{6}\right)$ and $\cos \left(-\frac{5\pi}{6}\right)$.

Using the formulas for supplements, we have

$$\sin \left(-\frac{5\pi}{6}\right) = \sin \left(\pi - \frac{5\pi}{6}\right) = \sin \left(\frac{\pi}{6}\right) = \frac{1}{2};$$
$$\cos \left(-\frac{5\pi}{6}\right) = \cos \left(\pi - \frac{5\pi}{6}\right) = -\cos \left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}.$$ 

Using the formula for negatives, we then get:

$$\sin \left(-\frac{5\pi}{6}\right) = -\sin \left(\frac{5\pi}{6}\right) = \frac{1}{2};$$
$$\cos \left(-\frac{5\pi}{6}\right) = \cos \left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}.$$
Problem 5. Find the length of the arc that subtends the angle 120° on a circle of diameter 8 in.

The radian measure of the angle is: \( \theta = 120 \times \frac{\pi}{180} = \frac{120\pi}{180} = \frac{2\pi}{3} \). The radius is half of the diameter, so \( r = 4 \) in. The length of the arc is then

\[ \ell = r\theta = 4 \cdot \frac{2\pi}{3} \text{ in} = \frac{8\pi}{3} \text{ in}. \]

Problem 6. Prove the identity:

\[ \sin^2 t (\csc^2 t - 1) = \cos^2 t. \]

\[ \text{LHS} = \sin^2 t \cdot \left[ \frac{1}{\sin^2 t} - 1 \right] = \sin^2 t \cdot \left[ \frac{1}{\sin^2 t} - \frac{\sin^2 t}{\sin^2 t} \right] = \sin^2 t \cdot \frac{1 - \sin^2 t}{\sin^2 t} = 1 - \sin^2 t = \cos^2 t = \text{RHS}. \]

Problem 7. The angle \( \theta \) is an acute angle in the right triangle shown in the figure.

Find all six trigonometric functions of \( \theta \).

Using Pythagoras we have

\[ \text{adj}^2 + \text{opp}^2 = \text{hyp}^2, \]

which gives \( \text{adj}^2 + 16^2 = 20^2 \). This means that

\[ \text{adj}^2 = 20^2 - 16^2 = 400 - 256 = 144, \]

so \( \text{adj} = \sqrt{144} = 12 \).

Using the well known formulas, we now get:

\[ \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{16}{20} = \frac{4}{5}; \]
\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{20} = \frac{3}{5}; \]
\[ \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{16}{12} = \frac{4}{3}; \]
\[ \cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{12}{16} = \frac{3}{4}; \]
\[ \sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{20}{12} = \frac{5}{3}; \]
\[ \csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{20}{16} = \frac{5}{4}. \]
Problem 8. Find the exact values of \( t \), in the interval \([0, 3\pi]\), which satisfy the equation

\[
\sin t = \frac{\sqrt{2}}{2}.
\]

Upon inspecting the graph:

we see that there are four solutions. Since the solution of \( \sin \theta = \frac{\sqrt{2}}{2} \) in the first quadrant is \( \theta = \frac{\pi}{4} \), it follows that the solutions \( t_1 \) and \( t_2 \) of the given equation, in the fundamental interval, must be chosen from the list:

\[
\frac{\pi}{4} \text{(Q.I)}; \quad \pi - \frac{\pi}{4} \text{(Q.II)}; \quad \pi + \frac{\pi}{4} \text{(Q.III)}; \quad 2\pi - \frac{\pi}{4} \text{(Q.IV)}.
\]

Since we want \( \sin t \) positive, the basic solutions have to be \( t_1 = \frac{\pi}{4} \) and \( t_2 = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \). The remaining solutions are then \( t_3 = t_1 + 2\pi = \frac{\pi}{4} + 2\pi = \frac{9\pi}{4} \) and \( t_4 = t_2 + 2\pi = \frac{3\pi}{4} + 2\pi = \frac{11\pi}{4} \).

Problem 9. Find the side labeled \( x \) in the right triangle:

We have

\[
\frac{x}{2\sqrt{3}} = \frac{\text{adj}}{\text{hyp}} = \cos 30^\circ = \frac{\sqrt{3}}{2},
\]

so we get

\[
2x = 2\sqrt{3} \cdot \sqrt{3} = 6.
\]

This gives \( x = \frac{6}{2} = 3 \).

Problem 10. Let \( \theta \) be an angle in standard position, such that the point \( P(-15, 17) \) is on its terminal side. Find all the six trigonometric functions of \( \theta \)

We are given \( x = -15 \) and \( y = 17 \), so we have

\[
r = \sqrt{x^2 + y^2} = \sqrt{(-15)^2 + 17^2} = \sqrt{225 + 289} = \sqrt{514}.
\]

Using the well known formulas, we now get:

\[
\sin \theta = \frac{y}{r} = \frac{17}{\sqrt{514}}; \quad \cos \theta = \frac{x}{r} = -\frac{15}{\sqrt{514}}; \quad \tan \theta = \frac{y}{x} = \frac{17}{15}; \quad \cot \theta = \frac{x}{y} = -\frac{15}{17};
\]

\[
\sec \theta = \frac{r}{x} = -\frac{\sqrt{514}}{15}; \quad \csc \theta = \frac{r}{y} = \frac{\sqrt{514}}{17}.
\]