Math100: Sample Exam 1a - Spring 07

1. Simplify avoiding negative exponents \((3x^{-3}y/z)^3/(3x^2/y^2)^2z^3\).

2. Suppose that the points \((-1, 2)\) and \((1, 5)\) lie at the ends of a diameter of a circle. Give the equation of the circle.

3. Rationalize the numerator of \(4 - 3\sqrt{2}/2 + 3\sqrt{2}\).

4. Find the equation of the line through the points: (a) \((3, -2), (3, 5)\), (b) \((-2, 1), (3, -5)\).

5. Find the equations of the lines through the point \((2, -3)\) which are parallel and perpendicular to the line \(5x - 2y = 3\).

6. What is the domain of \(x^3 - 4x/(x^3 - 3x^2 + x - 3)\)?

7. Simplify \(x^4 + 8x^3 + 16x^2/x^3 + 2x^2 - 8x\).

8. Use your calculator to graph \(y = x^5 - 10x^3 + 5x\). Find the coordinates of the intercepts to 4 decimal places.

9. For \(f(x) = 3x^2 - 6x\) find and simplify \(f(1 + h) - f(1)/h\).

10. Simplify \(1 - 3/3 - 2/(x + 2 - x + 1)\).

11. Write the polynomial in standard form \((x^2 - x + 1)(x^2 + x + 1)(x + 1)(x - 1)\).

12. Simplify \(1/x + 2 - 4/x^2 - 2x - 3 + 5/x^2 - x - 6\).

13. Factor as far as possible
   (a) \(25x^2z^2 - 4x^4\), (b) \(6x^2 - x - 15\), (c) \(3(x + 1)^3 + 6(x + 1)^2 + 3(x + 1)\), (d) \(3x^3 - 5x^2 - 3x + 5\).

14. Extract squares and simplify (a) \(7\sqrt{28} + 2\sqrt{63} - 3\sqrt{112}\).
   (b) \(\sqrt{75}y^{14} - \sqrt{2}y^3 \cdot 6y^9\) assuming that \(y\) is positive. What if \(y\) is negative?

15. Simplify using only positive rational exponents (assume \(x, y\) positive).
   (a) \(\sqrt[3]{9x^3y^2}/\sqrt[3]{xy^2}\)
   (b) \(7\sqrt[3]{y^3} \sqrt[3]{y} \sqrt[3]{y}\)
Math100: Sample Exam 1b - Spring 07

1. Simplify avoiding negative exponents \( \left( \frac{(3a^3b^{-3})^2b^2c}{(9a/b^2)^3bc^{-2}} \right)^3. \)

2. Give the equation of the circle which has the same center as the circle \((x+2)^2+(y-1)^2 = 9\) but passes through the point \((2, -3)\).

3. Rationalize the denominator of \( \frac{\sqrt{7} - \sqrt{2}}{\sqrt{7} + \sqrt{2}}. \)

4. Find the equation of the line which passes through the points \((-1, 2)\) and \((3, 8)\). Find the equation of the line which is perpendicular to this line and passes through \((2, -3)\).

5. Suppose that an $18,000 car is worth $3000 after 8 years and that the depreciation in value is linear. Give a linear function representing the value of the car after \(x\) years and estimate the value after 5 years.

6. What is the domain of \( \frac{x}{3x^2 + 10x - 8} \)?

7. Simplify \( \frac{x^3-x}{x^2-x-6} \div \frac{x^2-4x+3}{x^3-2x^2+x}. \)

8. Use your calculator to graph \( y = x^3 - 60x^2 + 30x + 60. \) Find the coordinates of the intercepts to 3 decimal places.

9. Suppose that \( f(x) = \frac{1}{x^2-2x}. \) Find the domain of \( f(1-x). \)

10. Simplify \( \frac{1}{x+2} - \frac{1}{x+3} \div \frac{1}{x^2+4x+4} - \frac{1}{x^2+5x+6}. \)

11. Write the polynomial in standard form \((x^3 + 3x - 2)(5x^2 - 3) + 2(x - 1)(x + 1).\)

12. Simplify \( \frac{2x+1}{x^2+x-6} + \frac{1}{x^2-5x+6}. \)

13. Factor as far as possible
   (a) \( x^2 - 6xy + 9y^2, \)  (b) \( x^4 - 5x^2 + 4, \)  (c) \( x^4(2x - 5)^4 + 3x^5(2x - 5)^3, \)  (d) \( 9x^3 - 27x^2 - 4x + 12. \)

14. Simplify (a) \( 3\sqrt[3]{16} - \sqrt[3]{250}. \)
   (b) \( \sqrt[4]{16y^{12}} - \sqrt[6]{8y^5} \) assuming that \( y \) is positive. What if \( y \) is negative?

15. Simplify using positive rational exponents (assume \( a, b > 0 \)):
   (a) \( \sqrt[2]{\frac{a}{b^2}} \sqrt[3]{a^2b} \)  (b) \( \sqrt[3]{\frac{a^2}{\sqrt{a}}}. \)