Math100: Sample Exam 2a - Spring 06

1. Write the complex numbers in standard form $a + ib$:
   (a) $\sqrt{-9} + \frac{1}{i}$,
   (b) $4(2i)(1-i)$,

2. Solve the quadratic equation $4x^2 - 12x = -9$.

3. Solve the equations (a) $\sqrt{x} - \sqrt{x-5} = 1$,
   (b) $\sqrt{x^2 + 4x - 2x} = 0$.

4. Solve $\frac{3y + 5}{y^2 + 5y} + \frac{y + 4}{y + 5} = \frac{y + 1}{y}$.

5. Solve the inequalities, giving the answer in interval notation
   (a) $-2 \leq 4 - 2x < 9$,
   (b) $4(2x - 2) + 2 > 3(x + 9) - x$.

6. Solve the inequality $|2x - 3| \leq 3$.

7. Graph the piecewise-defined function $f(x) = \begin{cases} 1, & x < 0, \\ 2 - x^2, & x \geq 0. \end{cases}$

8. Find the intercepts and vertex for $y = -x^2 + 12x - 25$.

9. (a) Is the function $f(x) = x^4 \sqrt{1 - 2x^2}$ even, odd or neither?
   (b) Describe the symmetries of the graph of $y^2 = 5x^4$.

10. Solve the linear equations for $x$
    (a) $3(2x + 2) + 9 = 2(3x - 7) - 9x + 5$,
    (b) $\sqrt{3}(x - 7) + 3 = 2(3x - \sqrt{3}) + 6$.

11. Given the graph of $y = f(x)$ graph $y = f(x)$
    (a) $f(x + 1)$,
    (b) $-2f(x)$,
    (c) $f(x) + 1$,

12. Use your calculator to find relative maxima & minima for $f(x) = \frac{1}{3}x^5 - 10x^3 + 75x$ and
    the intervals where $f(x)$ is decreasing (give answers to 3 decimal places).

13. A farmer has 1600ft of fencing to create a block eight
    cattle pens as shown.
   (i) Express the area enclosed as a function of $x$
   (ii) What width $x$ maximizes the enclosed area?
1. Write the complex numbers in standard $a + ib$ form: 
   (a) $i^{26} + i^{27} + i^{28}$, 
   (b) $\frac{\sqrt{5} + 3i}{1 - i}$.

2. Solve the quadratic equation $x^2 = 3x - 5$.

3. Solve the equation $\frac{3}{x+2} + \frac{2}{x} = \frac{4x - 4}{x^2 - 4}$.

4. Solve the equation $\sqrt{x} - \sqrt{3x} - 3 = 1$.

5. Solve the inequality $x - 4 < 2x - 5 \leq 2 - 3x$, giving the answer in interval notation.

6. Solve the inequality $|3x - 4| \geq 5$.

7. Graph the piecewise-defined function $f(x) = \begin{cases} -2x, & x \leq -1, \\ x + 2, & x > -1. \end{cases}$

8. Determine whether $f(x) = -2x^2 + 8x - 9$ has a maximum or minimum and find that value. Where is $f(x)$ increasing? Where is it decreasing? What are the intercepts?

9. (a) Describe the symmetries of the graph of $x^2 - 3y^3 = 1$.
   (b) Is the function $f(x) = x^3 + \frac{1}{x^3}$ even, odd or neither?

10. Given $f(x) = x + 2$ and $g(x) = \sqrt{x + 1}$, find (a) $(f + g)(4)$, (b) $(fg)(5)$, (c) $\left(\frac{f}{g}\right)(x)$.

11. Given the graph of $y = f(x)$ graph
    (a) $f(x - 1)$,
    (b) $f(2x)$,
    (c) $\frac{1}{2}f(-x)$.

12. Use your calculator to find the intervals where $f(x) = x^4 - 5x^2 + x + 2$ is increasing (give answers to 3 decimal places).

13. A section of gutter is to be made from a 130 by 16 inch rectangular sheet of metal by bending up a strip of width $x$ along each long side to form a U-shape.
    (i) Express the volume of the gutter as a function of $x$.
    (ii) What width $x$ maximizes the gutter’s volume?