1. (10pts) Factor into polynomials with real coefficients.
(a) \( x^3 - 3x^2 - 7x + 21 = \)
(b) \( 6x^2 - 7x - 20 = \)

2. (10pts) Perform the operations and simplify your answer into lowest terms.
(a) \( \frac{x}{x^2 + 11x + 30} - \frac{5}{x^2 + 9x + 20} = \)
(b) \( \frac{a^2 - a - 2}{a^2 - a - 6} \div \frac{a^2 - 2a}{2a + a^2} = \)

3. (8pts) Rationalize the denominator: \( \frac{5}{y - s\sqrt{x}} = \)

4. (8pts) Write an equation for the circle which has center \((-5, 6)\) and passes through the point \((1, 7)\).
5. (10pts) For the function \( f(x) = \frac{5x - 3}{2x + 1} \), find the inverse function \( f^{-1}(x) \).

6. (10pts) Write the slope-intercept equation of the line that has the same \( y \)-intercept as the line \( L_1 \) and is perpendicular to the line \( L_1 \) if \( L_1 \) is described by the equation \( 2y - 3x = 12 \).

7. (12pts) For the function \( f(x) = x^3 - 2500x + 100 \):
   (a). Use your calculator to graph the function and record your window settings.
   (b). List all relative maxima/minima and plot them on the graph (record major keys of your calculator used to get your answers).
   (c). List all intervals on which the function is increasing or decreasing.

8. (8pts) For the function \( f(x) = -5x^7 - 6x^3 - 2x \):
   (a). Test whether the function is even or odd.
   (b). What kind of symmetry does the graph of the function possess?
9. (6pts) Describe a sequence of transformations on the graph of the function \( f(x) = \log_3(x) \) to get the graph of the function \( g(x) = \log_3(x + 2) - 5 \).

10. (10pts) A rancher wants to create three rectangular corrals as shown in the picture and he has a total of 240 meters of chain-link fence to be used for all the walls. Determine the dimensions of each side so the largest possible total area can be enclosed.

11. (10pts) Solve the equations.
   (a) \( \log_4(x + 3) + \log_4(x - 3) = 2 \)
   (b) \( e^{x} - 6e^{-x} = -1 \)

12. (12pts) Given the following functions, describe the behavior of the graphs as \( x \to -\infty \) and \( x \to \infty \) by drawing one of the following graphs in the space provided for each function.

   a). \( f(x) = -2003x^{1000} + 1000x^{999} + 33333x - 99999 \) has the behavior ______
   b). \( f(x) = 10000x^{9999} - 99999x^{1000} + 999999 \) has the behavior ______
   c). \( f(x) = 99999x^{10000} - 100000x^{9999} - 99999 \) has the behavior ______
   d). \( f(x) = -20000x^{19999} - 19999x^{1000} + 99999 \) has the behavior ______
13. (10pts) Find all zeros of the polynomial function $f(x) = x^3 + 2x^2 - 2x + 3$ by doing the following:
(a). Use the rational zero test to find a rational zero.

(b). Use the zero you found in (a) and the factor theorem to find all other zeros.

14. (12pts) For the rational function $f(x) = \frac{x^2 - 196}{x + 4}$, do the following:
(a). Find all asymptotes (vertical, horizontal/slanted).

(b). Find all $x$-intercepts and $y$-intercepts.

(c). Sketch the graph of the function using the facts in (a) and (b).

15. (10pts) Express the logarithm $\log_a(\sqrt[3]{\frac{y^3 z^2}{x^4}})$ in terms of $\log_a(x)$, $\log_a(y)$, and $\log_a(z)$.

16. (12pts) Suppose that $10,000 is invested with an annual interest rate of 2.3% compounded continuously.
(a). Find the exponential function for the balance at the end of $t$ years

(b). Compute the balances at the end of 10 years and 30 years.

(c). What is the doubling time?
17. (10pts) Solve the system of linear equations using Gaussian elimination without using a calculator.

\[
\begin{align*}
\begin{cases}
x + y - 3z &= 4 \\
4x + 5y + z &= 1 \\
2x + 3y + 8z &= 2
\end{cases}
\end{align*}
\]

18. (10pts) Given matrices

\[
A = \begin{bmatrix}
-1 & 0 & 7 \\
3 & a & 2
\end{bmatrix}
\quad \text{and} \quad
B = \begin{bmatrix}
b & 4 \\
-4 & 0 \\
1 & 3
\end{bmatrix}
\]

perform the matrix operation:

(a). \(AB = \)

(b). \(BA = \)

19. (12pts) For the system of linear equations

\[
\begin{align*}
\begin{cases}
x + 2y + 3z &= -1 \\
2x - 3y + 4 &= 2 \\
-3x + 5y - 6z &= 4
\end{cases}
\end{align*}
\]

(a). Write the coefficient matrix \(A\) of the system and matrices \(X\) and \(B\) such that the system is equivalent to the matrix equation \(AX = B\).

(b). Use your calculator to find the inverse matrix \(A^{-1}\).

(c). Write the solutions of the given system of linear equations in terms of \(A^{-1}\) and \(B\).

20. (10pts) Decompose into partial fractions for \(\frac{2x + 1}{x^2 - x - 6}\).