Review Questions for Final Exam
Math 100 - Spring 04

Disclaimer: This is not a practice exam. It is meant to reflect the sort of material you will be expected to know on the exam, not to indicate the actual questions or length of the exam.

1. Perform the divisions: (a) \((x^4 - 1) ÷ (x - 2)\) (b) \((3x^3 - 2x^2 + 3x + 2) ÷ (x^2 + x - 1)\).

2. For \(f(x) = 2x^4 - 7x^3 + 11x^2 - 28x + 12\) find (a) the rational zeros, (b) the remaining zeros, and (c) factor \(f\) completely (use complex numbers if necessary).

3. Factor into linear factors (using complex numbers where necessary
   a) \(15x^2 - 19x + 6\)  (b) \(x^6 - 3x^4 - 10x^2\)  (c) \(x^4 - 9\).

4. Give the slope intercept form of the equation of the straight line through the point (1, -3) and (a) perpendicular to the line \(3x - 4y = 11\), (b) through the point (3, -5).

5. Solve the inequalities (a) \(x^4 < 4x^2\),  (b) \(\frac{2x}{x+3} \leq 1\),  (c) \(\frac{x}{x+3} < \frac{x+1}{x-1}\).

6. Solve the equations
   (a) \(\frac{5}{x-1} = \frac{3}{x-2} + \frac{1}{3x^2 - 3x + 2}\).
   (b) \(\frac{5}{x^2 - 4} = \frac{3}{x^2 - 3x + 2} + \frac{1}{x^2 + x - 2}\).
   (c) \(\frac{5}{x-1} = \frac{3}{x-2} + 1\).

7. \(f(x) = \frac{x^3 - 4x}{x^2 - 1}\).

Find the (a) intercepts, (b) asymptotes, (c) symmetry. (d) Where is \(f\) positive? (e) Where is \(f\) negative? Use this information to sketch the graph.

8. Find the domain, intercepts, asymptotes (all sorts), and test for symmetry
   a) \(f(x) = \frac{x^2 - 3x}{x^2 - 4}\),  b) \(g(x) = \frac{x^2 - 5x - 6}{x + 2}\),  c) \(h(x) = \frac{x}{x^2 - 3}\).

9. Solve the inequalities (a) \(|x - 5| \leq 7\),  (b) \(|2x - 3| > 5\).

10. Find the (real) solutions of (a) \(\sqrt{2x^3 + x} = 6\),  (b) \(|2x + 1| = 2 - x\).

11. Simplify (a) \(\frac{x - 1}{5} - \frac{x + 1}{6}\), (b) \(\frac{1}{x^2 + 2x} + \frac{1}{x^2 + 5x + 6}\).

12. Simplify using only positive rational exponents (you may assume \(x, y, z\) are positive)
   (a) \(\sqrt[3]{\frac{x^3 y}{\sqrt[3]{x^4 y^2}}}\),  (b) \(\frac{x^5 y^2 / z^2}{(xy^3)^2 (z/y)^3}\),  (c) \(y \sqrt[4]{y \sqrt[4]{y}}\).
13. (a) A company’s profit (in millions of Euros) from producing $x$ tons of its product is given by $P = 30x - 5 - 3x^2$. How much should it produce to maximize profits?
(b) Find the vertex of the parabola $y = -2x^2 - 8x + 5$ and say whether it is a maximum or a minimum. Find the intercepts.

14. Given the graph of $y = f(x)$ graph
(a) $f(x - 2)$,
(b) $f(x + 2)$,
(c) $f(x) + 2$,
(d) $f(-x)$,
(e) $-f(x)$,
(f) $2f(x)$,
(g) $f(2x)$,
(h) $-2f(x)$,
(i) $\frac{1}{2}f(-x)$.

15. (a) Write $\ln\left(\sqrt[3]{x^2} \cdot z^3/y^4\right)$ as a sum and difference of logs (assume $x, y, z$ positive).
(b) Write $2 \log_5(x + 5) - \frac{1}{2} \log_5 x + 7 \log_5 e$ as a single log.
(c) If $\ln a = 3$ and $\ln b = 5$ what is $\log_5 b$?

16. Solve (a) Solve $\log_3(3x - 8) = 2$.
(b) Solve $\log_3 x - \log_3(x - 8) = 2$.
(c) $\log_3 x + \log_3(x - 4) = 1$.

17. Solve the equations (a) $5^{3x-7} - 2 = 0$, (b) $2e^{2x+3} - 5 = 0$.

18. What are the domains of $f(x) = 2/x$ and $g(x) = \sqrt{x - 3}$? What are $f \circ g$ and $g \circ f$?

19. Find the inverse $f^{-1}(x)$ for the following functions or say why it does not exist.
(a) $f(x) = \frac{1}{2}(x^2 - 1)$, (b) $f(x) = \frac{3}{x - 5}$, (c) $f(x) = \sqrt{2x + 1}$.

20. (a) You deposit $1000 in a bank account offering an annual interest rate of 4.33% compounded continuously. How much will be in the account after 5 years? How many years before you have $2000 in the account?
(b) An artifact contains only 40% of its original carbon-14. If the half-life of carbon-14 is 5750 years, how old is the artifact?

21. Give the augmented matrix of the system of linear equations. Find (by calculator!) the reduced row echelon form and hence solve the system if possible.

\[
\begin{align*}
(a) & \quad 3x - 2y + z + 5w = -3 \\
& \quad 3x - 2y + z = 5 \quad \text{ (b) } \quad 3x - 2y + z + 3w = -3 \\
& \quad x + y - z + 3w = -3 \quad 3x - y - z = 7 \\
& \quad 2x + 2y - 3z - w = -2 \quad 9x - y - z = 9. \\
& \quad 5x + y - 5z + 2w = -10.
\end{align*}
\]

22. Give the partial fraction decomposition of (a) $\frac{1}{9x^2 - 9x + 2}$ (b) $\frac{2x^2 - 6x + 2}{x^3 - 3x^2 + 2x}$.
Solutions

1. (a) Quotient: $x^3 + 2x^2 + 4x + 8$, Remainder: 15, (b) Quotient: $3x - 5$, Remainder: $11x - 3$.
2. (a) $x = 3, \frac{1}{2}$, (b) $x = \pm 2i$, (c) $f(x) = (x - 3)(2x - 1)(x + 2i)(x - 2i)$.
3. (a) $(5x - 3)(3x - 2)$, (b) $x^2(x - \sqrt{5})(x + \sqrt{5})(x - \sqrt{2}i)(x + \sqrt{2}i)$, (c) $(x - \sqrt{3})(x + \sqrt{3})(x - \sqrt{3}i)(x + \sqrt{3}i)$.
4. (a) $y = -\frac{4}{3}x - \frac{5}{3}$, (b) $y = -x - 2$.
5. (a) $(-2, 0) \cup (0, 2)$, (b) $(-3, 3]$, (c) $(-3, -\frac{3}{5}) \cup (1, \infty)$.
6. (a) $x = 4$, (b) $x = 9$, (c) $x = \frac{5}{2} \pm \frac{\sqrt{11}}{2}i$.
7. (a) $(0, 0)$, $(2, 0)$, $(-2, 0)$, (b) vert. $x = 1, x = -1$, slant $y = x$.
   (c) odd.
   (d) $(-2, -1) \cup (0, 1) \cup (2, \infty)$.
   (e) $(-\infty, -2) \cup (-1, 0) \cup (1, 2)$.
8. (a) $x \neq \pm 2, (0, 0), (3, 0)$, $x = 2, x = -2, y = 1$, no symm.
   (b) $x \neq -2, (0, -3), (-1, 0), (6, 0)$, $x = -2, y = x - 7$, no symm.
   (c) $x \neq \pm \sqrt{3}, (0, 0), x = \sqrt{3}, x = -\sqrt{3}, y = 0$, odd.
9. (a) $[-2, 12]$, (b) $(-\infty, -1) \cup (4, \infty)$.
10. (a) $x = 3$, (b) $x = \frac{1}{3}, -3$.
11. (a) $\frac{-x}{x + 1}, x \neq -5, 0, 1$,
    (b) $\frac{(2x + 3)(x - 3)}{3(x + 2)}, x \neq 0, \pm 3$.
12. (a) $x^{\frac{7}{10}}y^{\frac{1}{20}},$ (b) $\frac{x^3}{yz^5}$, (c) $y^{4/3}$.
13. (a) 5 tons (70 million euros) (b) Maximum: $(-2, 13)$. Intercepts: $(0, 5), \left(-2 \pm \frac{\sqrt{11}}{2}, 0\right)$.
14. (a) $f(x - 2)$ (b) $f(x + 2)$ (c) $f(x) + 2$
15. (a) \( \frac{2}{3} \ln x + 3 \ln z - 4 \ln y. \)
    (b) \( \log_5 \left( \frac{e^{7(x+5)^2}}{\sqrt{x}} \right) . \)
    (c) \( x = \frac{5}{3}. \)
16. (a) \( x = \frac{17}{3} \)  (b) \( x = 9 \)  (c) \( x = 5. \)
17. (a) \( x = \frac{1}{3} \left( \frac{\ln 2}{\ln 5} + 7 \right) \approx 2.4769. \)  (b) \( x = \frac{1}{2} \left( \ln(\frac{5}{2}) - 3 \right) \approx -1.0419. \)
18. \( x \neq 0, x \geq 3. \)  (f \( \circ \) g)(x) = \( \frac{2}{\sqrt{x-3}}, \)  (g \( \circ \) f)(x) = \( \sqrt{\frac{2}{x-3}}. \)
19. (a) No inverse (graph fails horiz. line test), (b) \( f^{-1}(x) = \frac{3}{x} + 5, \)  (c) \( f^{-1}(x) = \frac{1}{2}(x^3-1). \)
20. (a) $1241.72$, 16 years.  (b) 7,601 years.
21. (a) \( \begin{pmatrix} 3 & -2 & 1 & 5 & -3 \\ 1 & 1 & -1 & 3 & -3 \\ 2 & 2 & -3 & -1 & -2 \\ 5 & 1 & -5 & 2 & -10 \end{pmatrix}, \)  \( \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \), \( x = 1, y = 2, z = 3, w = -1. \)
   (b) \( \begin{pmatrix} 3 & -2 & 1 & 5 \\ 1 & 1 & -1 & 7 \\ 9 & -1 & -1 & 9 \end{pmatrix}, \)  \( \begin{pmatrix} 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -\frac{2}{5} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \), no solution.
22. (a) \( \frac{1}{3x-2} - \frac{1}{3x-1}, \)  (a) \( \frac{1}{x} - \frac{1}{x-2} + \frac{2}{x-1}. \)