Review Questions for Exam III
Math 100 - Spring 04

Disclaimer: This is not a practice exam. It is meant to reflect the sort of material you will be expected to know on the exam, not to indicate the actual questions or length of the exam.

1. Find the quotient and remainder when
   (a) \(x^5 - 20\) is divided by \(x - 2\).
   (b) \(2x^4 - 7x^3 - 3x + 2\) is divided by \(x^2 - 3x + 1\).

2. (a) Give a polynomial of degree 4 with rational coefficients, a zero of multiplicity 2 at -1 and a zero at \(3 + \sqrt{2}\).
   (b) Give a polynomial of degree 4 with real coefficients and zeros \(1 + 2i\), \(3 - 4i\).

3. \(f(x) = x^4 + 3x^3 - x - 3\).
   (a) Find the rational roots of \(f\).
   (b) Factor \(f\) into polynomials with real coefficients.
   (c) Factor \(f\) into linear factors (use complex numbers as necessary).

4. (a) Find the rational roots of \(f(x) = 2x^4 - x^3 - 13x^2 + 5x + 15\).
   (b) Find the remaining roots.

5. For the following polynomials:
   (i) Write as a product of linear factors. (ii) Find the zeros and their multiplicities.
   (a) \(f(x) = x^4 + 5x^3 + 6x^2\), (b) \(g(x) = (x^4 + x^2 - 12)^2\)

6. Solve the inequalities
   (a) \(x^4 < 4x^2\), (b) \(x^3 \geq 6x^2 - 8x\).

7. Solve the inequalities (a) \(\frac{x}{x - 5} \leq 2\), (b) \(\frac{x}{x + 3} \leq \frac{x + 1}{x - 1}\).

8. (i) \(f(x) = \frac{x^3 - 4x}{x^2 - 1}\), (ii) \(g(x) = \frac{2x^2 - 10x - 12}{x^2 - 4}\),
   Find the (a) intercepts, (b) asymptotes, (c) symmetry. (d) Where is the function positive? Use this information to sketch the graph.

9. Find the asymptotes (all sorts):
   a) \(f(x) = \frac{x^2 + x - 6}{x + 2}\), b) \(g(x) = \frac{x}{x^2 - 3}\), c) \(h(x) = \frac{x^6 - 3x^2 - 4}{x^4 + 4}\).

10. Describe the behavior as \(x \to -\infty\):
    (a) \(f(x) = 3x^4 + 2x^3 - 2000x^2 + 5x - 3\).
    (b) \(g(x) = 10 - 2000x + 5x^3\).

11. A polynomial \(f\) has degree 23. How many \(x\)-intercepts can it have? How many turning points? If \(f(x) \to \infty\) as \(x \to -\infty\) what happens as \(x \to \infty\)?
Solutions

1. (a) Quotient $x^4 + 2x^3 + 4x^2 + 8x + 16$, Remainder 12.
   (b) Quotient $2x^2 - x - 5$, Remainder $-17x + 7$.

2. (a) e.g. $(x + 1)^2(x - 3 - \sqrt{2})(x - 3 + \sqrt{2}) = x^4 - 4x^3 - 4x^2 + 8x + 7$.
   (b) e.g. $(x - 1 - 2i)(x - 1 + 2i)(x - 3 + 4i)(x - 3 - 4i) = x^4 - 8x^3 + 42x^2 - 80x + 125$.

3. (a) $x = -3, 1$.
   (b) $f = (x + 3)(x - 1)(x^2 + x + 1)$.
   (c) $f = (x + 3)(x - 1) \left( x - \left( -\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \right) \left( x - \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \right)$.

4. (a) $x = -1, \frac{3}{2}$.
   (b) $x = \sqrt{5}, -\sqrt{5}$.

5. (a) (i) $x^2(x + 3)(x + 2)$, (ii) $x = 0$ (mult 2), $x = -2$ (mult 1), $x = -3$ (mult 1).
   (b) (i) $(x + 2i)^2(x - 2i)^2(x + \sqrt{3})^2(x - \sqrt{3})^2$, (ii) $x = \pm 2i, \pm \sqrt{3}$ (all mult 2).

6. (a) $(-2, 0) \cup (0, 2)$. (b) $[0, 2] \cup [4, \infty)$.

7. (a) $(-\infty, 5) \cup [10, \infty)$. (b) $(-3, -\frac{3}{2}) \cup (1, \infty)$.

8. (i) (a) $(0, 0)$, $(2, 0)$, $(-2, 0)$.
   (b) Vertical: $x = 1$, $x = -1$. Oblique: $y = x$.
   (c) Odd.
   (d) $(-2, -1) \cup (0, 1) \cup (2, \infty)$.

   (ii) (a) $(0, 3)$, $(6, 0)$, $(-1, 0)$.
   (b) Vertical: $x = 2$, $x = -2$. Horizontal: $y = 2$.
   (c) None.
   (d) $(-\infty, -2) \cup (-1, 2) \cup (6, \infty)$.

   (b) Vertical: $x = \sqrt{3}$, $x = -\sqrt{3}$. Horizontal: $y = 0$.
   (c) None.

10. (a) $f(x) \to \infty$. (b) $g(x) \to -\infty$.

11. At most 23 $x$-intercepts. At most 22 turning points. As $x \to \infty$, $f(x) \to -\infty$. 