Review Questions for Exam 3

Disclaimer: This is not a practice exam. It is meant to reflect the sort of material you will be expected to know on the exam, not to indicate the actual questions or length of the exam.

1. \( f(x) = -2x^2 + 4x + 3. \)
   (a) Find the axis of symmetry and vertex.
   (b) Find the intercepts.
   (c) Sketch the graph of \( f \) and write \( f \) in standard form.

2. Find the quotient and remainder when \( 2x^4 - 7x^3 - 3x + 2 \) is divided by \( x^2 - 3x + 1. \)

3. Give a polynomial of degree 4 with real coefficients, a root of multiplicity 2 at \(-1\) and a root at \( 3 + \sqrt{2} i. \)

4. \( f(x) = \frac{3x^2 - x - 2}{x^2 - 5x + 6}, \quad g(x) = \frac{x^2}{x^4 + 1}, \quad h(x) = \frac{x^3 + 2x^2 + 1}{x^2 - 2x + 1}. \)
   Identify the asymptotes (vertical, horizontal and slant) and test for symmetry.

5. \( f(x) = \frac{x^3 - 4x}{x^2 - 1}. \)
   Find the (a) intercepts, (b) asymptotes, (c) symmetry. (d) Where is \( f \) positive? (e) Where is \( f \) negative? Use this information to sketch the graph.

6. \( f(x) = x^4 + 3x^3 - x - 3. \)
   (a) Find the rational roots of \( f. \)
   (b) Factor \( f \) over the reals.
   (c) Factor \( f \) into linear factors (use complex numbers as necessary).

7. (a) Find the rational roots of \( f(x) = 2x^4 - x^3 - 13x^2 + 5x + 15. \)
   (b) Find the remaining roots.

8. \( f(x) = x^4 + 5x^3 + 6x^2 \)
   (a) Find the zeros and their multiplicities.
   (b) Find where \( f \) is positive.
   (c) Determine the behavior of \( f \) as \( x \to \infty \) and as \( x \to -\infty. \)
   (d) Sketch the graph of \( f. \)

9. (i) A company sells 5000 lbs of its product if it charges $10 per pound. For every $1 increase in price the demand will decrease by 100 lbs. Suppose that the price is increased by $x.
   (a) Write the new price and demand in terms of \( x. \) Write the revenue in terms of \( x. \)
   (b) What choice of \( x \) maximizes the revenue?
(ii) A farmer has 1000ft of fencing to enclose four adjacent corrals as shown. What width $x$ maximizes the enclosed area?

10. Find the constant $c$ such that

$$\frac{x^3 - 2x^2 + cx - c}{x - 3} = \text{polynomial} + \frac{3}{x - 3}$$

(i.e. the remainder when $x^3 - 2x^2 + cx - c$ is divided by $(x - 3)$ is 3).

11. Give the equation of the parabola with vertex $(-3, 11)$ which passes through $(-2, 7)$.

**Solutions**

1. (a) $x = 1$, $(1, 5)$.
   (b) $x$-intercepts: $(1 + \frac{1}{2}\sqrt{10}, 0), (1 - \frac{1}{2}\sqrt{10}, 0)$; $y$-intercept: $(0, 3)$.
   (c) $y = -2(x - 1)^2 + 5$.

2. Quotient $2x^2 - x - 5$, Remainder $-17x + 7$.

3. e.g. $(x + 1)^2(x - 3 - \sqrt{2}i)(x - 3 + \sqrt{2}i) = x^4 - 4x^3 + 16x + 11$.

4. $f : x = 3, x = 2, y = 3$, no slant, no symmetry.
   $g : y = -2(x - 1)^2 + 5$.
   $h : x = 1$, no horiz., $y = x + 4$, no symmetry.

5. (a) $(0, 0), (2, 0), (-2, 0)$,
   (b) vert. $x = 1, x = -1$, slant $y = x$,
   (c) odd.
   (d) $(-2, -1) \cup (0, 1) \cup (2, \infty)$.
   (e) $(-\infty, -2) \cup (-1, 0) \cup (1, 2)$.

6. (a) $x = -3, 1$.
   (b) $f = (x + 3)(x - 1)(x^2 + x + 1)$.
   (c) $f = (x + 3)(x - 1)(x - \frac{1}{2}(1 + \sqrt{3}i))(x - \frac{1}{2}(1 - \sqrt{3}i))$.

7. (a) $x = -1, \frac{3}{2}$.
   (b) $x = \sqrt{5}, -\sqrt{5}$.

8. (a) $x = 0$ (mult 2), $x = -2$ (mult 1), $x = -3$ (mult 1).
   (b) $(-\infty, -3) \cup (-2, 0) \cup (0, \infty)$.
   (c) $+\infty, +\infty$.

9. (i) $p = 10 + x, d = 5000 - 100x, R = -100x^2 + 4000x + 50000.
   (b) $x = \$20$.
   (ii) $x = 100$ ft.

10. $c = -3$.

11. $y = -4(x + 3)^2 + 11$.  