Section 4.4/4.5 Logarithmic and Exponential Equations

Definition of Logarithm:
\[ b^x = y \text{ if and only if } \log_b(y) = x \]

Properties of Logarithms:
\[
\begin{align*}
\cdot \log_b(xy) &= \log_b(x) + \log_b(y) \\
\cdot \log_b\left(\frac{x}{y}\right) &= \log_b(x) - \log_b(y) \\
\cdot \log_b(x^n) &= n \log_b(x)
\end{align*}
\]

Section 4.5:

- If \( a^x = a^y \), then \( x = y \)

Ex) Solve for \( x \):
\[
2^x = 32 \\
2^x = 2^5 \quad \text{So } x = 5
\]

- If \( \log_b(x) = \log_b(y) \), then \( x = y \).

Ex) Solve for \( x \): \( \log(x+5) - \log(x-3) = \log 2 \).
\[\text{(Set up only)} \quad \log\left(\frac{x+5}{x-3}\right) = \log 2 \]
\[\text{So } \frac{x+5}{x-3} = 2 \; \text{ solve for } x.\]
Exponential & Logarithmic Equations

... until now, we've solved

a) linear equations
b) quadratic equations → factored/AF/CD
  c) polynomial equations → factoring
d) Rational Equations → find LCD, etc.

1) Exponential Equations:

Ex) Solve $2e^x = 10$

Exponential Form → $e^x = 5$

Convert to Logarithmic Form (use definition of logarithm)

Answer in exact form

2) Logarithmic Equations

Ex) Solve: $6 \cdot \log_3 \left( \frac{1}{3}x \right) = 18$

Logarithmic Form → $\log_3 \left( \frac{1}{3}x \right) = 3$

Convert to exponential form

$3^3 = \frac{1}{3}x$

$x = 81$ Solution
More Examples:

3. Solve for $x$: \( \log(x+5) - \log(x-3) = \log 2 \)

\[
\log \left( \frac{x+5}{x-3} \right) = \log 2
\]

Then: \( \frac{x+5}{x-3} = 2 \)

Rational Equation

\( x+5 = 2(x-3) \)

\( x+5 = 2x-6 \)

\( x = 11 \)

Check Answer: \( \frac{12}{1} \)

4. Solve for $x$: \( \ln(x) + \ln(x+2) = 1 \)

\( \ln(x^2+2x) = 1 \) logarithmic form

\( e^1 = x^2+2x \)

Convert to exponential form

\( 0 = x^2+2x-e \) Quadratic Equation

Use Q. Formula: \( a=1, \ b=2, \ c=-e \)

\[
\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4+4e}}{2}
\]

\[
x = -1 + \sqrt{1+e} \approx 0.9283 \]

\[
x = -1 - \sqrt{1+e} \approx -2.9283
\]

Domain of \( \ln(x) \)?

\( x > 0 \)
5. Solve for $q$:

$$8 \cdot 2^q = 512$$

$$2^q = 64$$

$$2^q = 2^6 \quad \Rightarrow \quad q = 6$$

6. Solve for $x$:

$$8 \cdot 3^x = 512$$

**Exp Form**

$$3^x = 64$$

**Convert to log Form:**

$$\log_3 (64) = x$$

**Alternate Method:** "Taking the logarithm of both sides"

$$3^x = 64$$

$$\log (3^x) = \log (64)$$

$$x \log 3 = \log 64$$

$$x = \frac{\log 64}{\log 3}$$

Answer

Could have used $\ln$:

$$x = \frac{\ln 64}{\ln 3}$$

Same thing
Ex) Solve using technology

\[ e^{7.2x} = 14.009 \]

\[ Y_1: e^{7.2x} \quad x_{\text{min}}: -1 \]
\[ Y_2: 14.009 \quad y_{\text{min}}: 0 \]
\[ x_{\text{max}}: 1 \quad y_{\text{max}}: 20 \]

2nd + \text{trace}
Pick intersect.

\[ x \approx 0.366625 \]

Algebraically: Solve \[ e^{7.2x} = 14.009 \] (exponential equation)

\[ \ln(14.009) = 7.2x \quad \therefore x = \frac{\ln(14.009)}{7.2} \]
Online HW: \( \ln 2 = 0.693 \quad \ln a = 1.29 \quad \ln 3 = 1.099 \quad \ln b = 2.874 \)

\[
 a) \quad \ln \left( \frac{3 \sqrt{a^2}}{b^5} \right) \\
\ln \left( \frac{a^2}{b^3} \right)^{1/3} = \ln \left( \frac{a^{2/3}}{b^{5/3}} \right) \\
= \ln (a^{2/3}) - \ln (b^{5/3}) \\
= \frac{2}{3} \ln a - \frac{5}{3} \ln b \\
= \frac{2}{3} (1.29) - \frac{5}{3} (2.874)
\]

(b) \( \ln (8a) = \)

\[
\ln (8) + \ln (a) = 3 \ln 2 + \ln a \\
= \ln (2.4) + \ln (a) = 3 \ln 2.4 + 1.29 \\
= \ln (2.242) + \ln (a) = \text{use calc}
\]

Reminder: Last E.C. Due today & Online HW 14 due Thurs @ 8 am.