Math100: Sample Exam 1a - Fall 05

1. Simplify avoiding negative exponents \(\frac{(3x^{-3}y/z)^3}{(3x^2/y^2)^2z^3}\).

2. Suppose that the points \((-1, 2)\) and \((1, 5)\) lie at the ends of a diameter of a circle. Give the equation of the circle.

3. Rationalize the numerator of \(\frac{4 - 3\sqrt{2}}{2 + 3\sqrt{2}}\).

4. Find the slope of the line between the points (a) \((3, -2), (3, 5)\), (b) \((-2, 1), (3, -5)\).

5. Suppose that the average cost of tuition was \$9,800 in 1998 and \$12,400 in 2002 and that the rise in costs is a linear function of time. Give a linear function for the cost \(x\) years after 1998 and use it to estimate the cost in 2005.

6. What is the domain of \(\frac{x^3 - 4x}{x^3 - 3x^2 + x - 3}\)?

7. Simplify \(\frac{x^4 + 8x^3 + 16x^2}{x^3 + 2x^2 - 8x}\).

8. Use your calculator to graph \(y = x^5 - 10x^3 + 5x\). Find the coordinates of the intercepts to 4 decimal places.

9. For \(f(x) = 3x^2 - 6x\) find and simplify \(\frac{f(1 + h) - f(1)}{h}\).

10. Simplify \(\frac{1 - \frac{3}{x + 2}}{\frac{2}{x + 2} - \frac{1}{x + 1}}\).

11. Write the polynomial in standard form \((x^2 - x + 1)(x^2 + x + 1)(x + 1)(x - 1)\).

12. Simplify \(\frac{1}{x + 2} - \frac{4}{x^2 - 2x - 3} + \frac{5}{x^2 - x - 6}\).

13. Factor as far as possible
   (a) \(25x^2z^2 - 4x^4\), (b) \(6x^2 - x - 15\), (c) \(3(x + 1)^3 + 6(x + 1)^2 + 3(x + 1)\), (d) \(3x^3 - 5x^2 - 3x + 5\).

14. Extract squares and simplify (a) \(7\sqrt{20} + 2\sqrt{45} - 3\sqrt{80}\).
   (b) \(\sqrt{75y^{10}} - \sqrt{2y^3} \cdot 6y^7\) assuming that \(y\) is positive. What if \(y\) is negative?

15. Simplify using only positive rational exponents (assume \(x, y\) positive).
   (a) \(\sqrt[7]{9x^3y^2}/\sqrt[5]{xy^2}\)
   (b) \(7\sqrt[\frac{3}{5}]{y^3 \sqrt[\frac{3}{7}]{y}}\)
1. Simplify avoiding negative exponents \( \left( \frac{(3a^3b^{-3})^2b^2c}{(9a/b^2)^3bc^{-2}} \right)^3 \).

2. Give the equation of the circle which has the same center as the circle \((x+2)^2+(y-1)^2 = 9\) but passes through the point \((2, -3)\).

3. Rationalize the denominator of \( \frac{\sqrt{7} - \sqrt{2}}{\sqrt{7} + \sqrt{2}} \).

4. Find the slope of the line between the points (a) \((-1, 2), (3, 8)\), (b) \((-2, 3), (1, 3)\).

5. Suppose that an $18,000 car is worth $3000 after 8 years and that the depreciation in value is linear. Give a linear function representing the value of the car after \(x\) years and estimate the value after 5 years.

6. What is the domain of \( \frac{x}{3x^2 + 10x - 8} \)?

7. Simplify \( \frac{x^3 - x}{x^2 - x - 6} \cdot \frac{x^2 - 4x + 3}{x^3 - 2x^2 + x} \).

8. Use your calculator to graph \( y = x^3 - 60x^2 + 30x + 60\). Find the coordinates of the intercepts to 3 decimal places.

9. Suppose that \( f(x) = \frac{1}{x^2 - 2x} \). Find the domain of \( f(1 - x) \).

10. Simplify \( \frac{1}{x+2} - \frac{1}{x+3} \frac{1}{x^2 + 4x + 4} - \frac{1}{x^2 + 5x + 6} \).

11. Write the polynomial in standard form \((x^3 + 3x - 2)(5x^2 - 3) + 2(x - 1)(x + 1)\).

12. Simplify \( \frac{2x + 1}{x^2 + x - 6} + \frac{1}{x^2 - 5x + 6} \).

13. Factor as far as possible (a) \( x^2 - 6xy + 9y^2 \), (b) \( x^4 - 5x^2 + 4 \), (c) \( x^4(2x - 5)^4 + 3x^5(2x - 5)^3 \), (d) \( 9x^3 - 27x^2 - 4x + 12 \).

14. Simplify (a) \( 3\sqrt[3]{16} - \sqrt[3]{256} \).
(b) \( \sqrt[12]{16y^2} - \sqrt[5]{8y^5} \) assuming that \( y \) is positive. What if \( y \) is negative?

15. Simplify using only positive rational exponents (assume \( a, b \) positive).
   (a) \( \sqrt[3]{\frac{a}{b^2}} \sqrt[3]{a^2b} \)  (b) \( \sqrt[3]{\frac{a^2}{\sqrt[3]{a}}} \).