1. Show that
   (a) $1 + x \leq e^x$, $\forall$ real $x$.
   (b) $e^x \leq 1 + 1.01x$, $\forall 0 \leq x \leq 0.01$.
   And use the results to show
   $$(1 + u)^n \leq 1 + 1.01nu \quad \text{if} \ 0 \leq nu \leq 0.01.$$ 

2. By constructing a fixed-point iteration, find the value of $x$ given by
   $$x = \sqrt{p + \sqrt{p + \sqrt{p + \cdots}}}$$
   where $p$ is a positive number. Prove the convergence of the fixed-point iteration.

3. Construct a near-minimax polynomial of degree $\leq 2$ for the function $g(t) = e^t$ on the interval $t \in [0, 1]$ and estimate its maximum error. You can express the result in terms of the exponential function. (hint: $\cos(n + 1)\theta = \cos \theta \cos n\theta - \cos(n - 1)\theta$)

4. Given the trapezoidal rule and Simpson’s rule as:
   $$\int_{x_0}^{x_0 + h} f(x) \, dx = \frac{h}{2} [f(x_0) + f(x_0 + h)] - \frac{h^3}{12} f^{(2)}(\xi),$$
   $$\int_{x_0}^{x_0 + 2h} f(x) \, dx = \frac{h}{3} [f(x_0) + 4f(x_0 + h) + f(x_0 + 2h)] - \frac{h^5}{90} f^{(4)}(\xi),$$
   (1) Describe the composite trapezoidal rule and composite Simpson’s rule for evaluating integrals $\int_a^b f(x) \, dx$ using $n$ subintervals.
   (2) Derive an estimate for the error in the composite trapezoidal rule in terms of the length of the subintervals into which $[a, b]$ is divided.
   (3) Derive the asymptotic error formula for the composite Simpson’s rule
   $$E_n(f) = -\frac{h^4}{180} [f^{(3)}(b) - f^{(3)}(a)],$$
   where $h = (b - a)/n$.

5. Let $U$ and $V$ be two $3 \times 3$ matrices such that
   $$UV = [w_{ij}] = \begin{bmatrix} w_{11} & w_{12} & 0 \\ w_{21} & w_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{ where } w_{11}w_{22} \neq w_{12}w_{21}.$$
Show that either the last row of $U$ is the zero row vector or the last column of $V$ is the zero column vector. (Hint: If $U$ is singular then there exists a nonzero 3-tuple vector $p$ such that $p^T U = 0$)

6. Let $A$ be an $n \times n$ symmetric real positive definite matrix. Show that there exist $2^n$ real lower-triangular matrices $L$ such that $A = LL^T$?

7. Assume that $w \in \mathbb{R}^n$, and that $\|w\|_2 = 1$. What are the eigenvalues, eigenvectors, and determinant of a Householder matrix $I - 2ww^T$?

8. By using the singular value decomposition, show that any square real matrix $A$ can be written as $A = QS$ where $Q$ is an orthogonal matrix and $S$ is a semipositive definite matrix.