1. A rectangular parallelepiped has sides 31 mm (millimeter), 42 mm and 53 mm, measured only to the nearest millimeter. Give a practical upper and lower bounds for the total surface area of the parallelepiped.

2. The computation of the sequence \( p_n = (1/3)^n \) is a stable mathematical problem. The sequence can also be generated by the following recurrence relation:

\[
p_0 = 1, p_1 = 1/3, p_n = \frac{5}{3}p_{n-1} - \frac{4}{9}p_{n-2}, n = 2, 3, \ldots
\]

Indicate with proof if this recurrence relation is stable.

3. Suppose that the equation \( f(x) = 0 \) can be rearranged as \( x = h(x) \) for some function \( h \in C^2 \).

   (1) Show that if, for some non-zero value of a parameter \( \omega \), the sequence generated by

   \[
x_{n+1} = x_n + \omega[h(x_n) - x_n]
   \]

   converges to a number \( \alpha \), then \( \alpha \) must be a fixed point of \( h \).

   (2) Taking \( g(x) = x + \omega[h(x) - x] \), deduce that the above iteration is second order if

   \[
   \omega = \frac{1}{1 - h'(\alpha)}, \quad h'(\alpha) \neq 1
   \]

   (3) Based on the discussion, can you suggest a practical way of choosing a suitable \( \omega \) during the iteration? (you do not need to give a proof for it)

4. Suppose \( f(x) \) have an \((n + 1)\)st derivative in \([a,b]\) and \( P_n(x) \) is the interpolation polynomial with respect to \( n + 1 \) distinct points \( x_i, i = 0, 1, \ldots, n, x_i \in [a,b] \) (i.e. \( P_n(x_i) = f(x_i) \)). Show that for any \( x \in [a,b] \), there exists a \( \xi = \xi(x) \), with

\[
\min(x_0, x_1, \ldots, x_n, x) < \xi < \max(x_0, x_1, \ldots, x_n, x),
\]

such that

\[
f(x) - P_n(x) \equiv R_n(x) = \frac{\omega_n(x)}{(n+1)!} f^{(n+1)}(\xi).
\]

where

\[
\omega_n(x) \equiv (x-x_0)(x-x_1)\ldots(x-x_n).
\]

5. Given the following theorem: a quadrature formula

\[
I_n\{f\} = \sum_{j=1}^{n} \alpha_j f(x_j)
\]

(1)