1. A function $v \in C^2(\Omega)$ is said to be subharmonic in $\Omega$ if $\Delta v \geq 0$ in $\Omega$.

Show that if $x_0 \in \Omega$ and $r < d(x_0, \partial \Omega)$, then

$$v(x_0) \leq |B(x_0, r)|^{-1} \int_{B(x_0, r)} v(\xi) \, d\xi.$$  

2. Solve by separation of variables

$$\begin{cases}
  u_t = u_{xx}, & \text{in } (0, 1) \times (0, \infty) \\
  u_x(0, t) = u_x(1, t) = 0 & \text{for } t > 0 \\
  u(x, 0) = u_0(x), & 0 < x < 1,
\end{cases}$$

where $u_0 \in L^2((0, 1))$. Verify that $u$ is a classical solution, and that $u(x, t) \to u_0$ in $L^2((0, 1))$ when $t \to 0$. In addition, $u(x, t) \to \int_0^1 u_0(x) \, dx$ uniformly when $t \to \infty$. Give a physical interpretation of this fact.

3. Use the Fourier transform to solve the problem

$$\begin{cases}
  u_{tt} - u_{xx} = h(x, t), & \text{in } \mathbb{R} \times (0, \infty) \\
  u(x, 0) = u_t(x, 0) = 0 & \text{in } \mathbb{R}.
\end{cases}$$

4. Suppose that $y, z \in C^\infty[-1, 1]$ and

$$(x^2 + 1)y'' + 2xy' + 3y = 0 \quad (x^2 + 1)z'' + 2xz' + 4z = 0 \quad y(-1) = 0 = y(1) \quad z(-1) = 0 = z(1)$$

Show that

$$\int_{-1}^1 y(x)z(x) \, dx = 0.$$  

5. Solve the initial value problem

$$yu_x + u_y = x \quad u(x, 0) = x^2$$

6. Given $\frac{dx}{dt} = \cos(x - t)$, $x(0) = 1$, show $x(t) > t$ for all $t > 0$. 

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