1. (i) Formulate maximum principle for Cauchy’s problem for parabolic equations.

(ii) Give an example of its applications without proof.

2. Solve the exterior Dirichlet problem

\[\begin{align*}
\Delta u &= 0 \quad \text{for } |x| > 1 \\
u(x) &= c \quad \text{for } |x| = 1 \\
x &\in \mathbb{R}^3.
\end{align*}\]

3. (i) State the Huygens Principle. How does it depend on the space dimension?

(ii) Define the energy integral \(E(t)\) for the wave equation

\[\begin{align*}
\frac{\partial u}{\partial t} &= c^2 \Delta u \\
u(0, x) &= \phi(x), \quad u_t(0, x) = \psi(x) \\
u(t, x)|_{x \in \partial \Omega} &= 0
\end{align*}\]

Show that the \(E'(t) = 0\).

4. (i) For \(u \in L^1(\Omega)\) define the weak derivative \(D_i u\). (1 \(\leq i \leq n\)) where \(\Omega \subset \mathbb{R}^n\) is a bounded domain.

(ii) Define the Sobolev space \(W^{2,p}(\Omega)\) where \(p \geq 1\) and give the definition of the norm \(\|u\|_{W^{2,p}}\).

5. A semidisc of radius \(a\) is well isolated from the surrounding medium. The temperature at \(ADB\) and segment \(AB\) are kept at \(u = c_2\) and \(u = c_1\) respectively. Find the stationary temperature distribution \(u\) in the semidisc. (See Fig. 1.)

6. Solve the following equation in the upper half-plane

\[\begin{align*}
\Delta u(x, y) &= 0, \quad -\infty < x < \infty, \quad 0 < y < \infty \\
u(x, 0) &= f(x)
\end{align*}\]

7. (i) Give the definition of the well posed problem in the sense of Hadamard.

(ii) Prove that the solution of the Dirichlet problem

\[\begin{align*}
\Delta u &= 0 \\
u|_{\partial \Omega} &= f(x)
\end{align*}\]

depends on \(f\) continuously.

8. Solve the following equation

\[\begin{align*}
xu_y - yu_x &= u \\
u(x, 0) &= h(x) \quad (x, y) \in \mathbb{R}^2
\end{align*}\]

where \(u = u(x, y), h\) is a given function.