1. Find the solution of the equation

\[ y \frac{\partial u}{\partial x} - x \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \]

such that \( u(x, y, z) = xy \) on the plane \( \{z = 0\} \).

2. Give an example of a function in some plane domain \( \Omega \subset \mathbb{R}^2 \) which belongs to \( W^{1,2}(\Omega) \), but does not belong to \( W^{2,2}(\Omega) \).

3. Find all distributional solutions of the equation \( x^2 \frac{d^2 y(x)}{dx^2} = 0 \).

4. Let \( u(x, t) \) be a sufficiently smooth solution of the problem

\[ \frac{1}{1 + t^2} u_t - \Delta u = e^u \]

in some region of space-time \( \mathbb{R}^n \times \mathbb{R} \) containing the cylinder

\[ Q = \{(x, t) \mid |x| \leq 1, \ 0 \leq t \leq 2\}. \]

Show that the minimum of \( u(x, t) \) in \( Q \) can be attained only on the set

\[ \Sigma = \{(x, t) \mid |x| \leq 1 \text{ and } t = 0, \text{ or } |x| = 1 \text{ and } 0 < t \leq 2\}. \]

5. Let \( u(x, t) \) be a 1-periodic in \( x \) finite energy solution of the equation

\[ \frac{\partial^2 u}{\partial t^2} + \mu \cdot (1 + t^2) \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, \]

where \( \mu \) is a positive constant. [Recall, that ”finite energy” implies that \( u(x, t) \) is a continuous function of \( t \) with values in \( W^{1,2}([0, 1]) \) and \( u_t(x, t) \) is a continuous function of \( t \) with values in \( L^2([0, 1]) \).]
Derive an energy estimate and use it to prove that
\[ \int_0^1 |u_t(x, t)|^2 \, dx \to 0 \]
as \( t \to +\infty \).

6. Consider the problem
\[
\begin{cases}
  u_{tt} - u_{xx} + (1 + t^2)u = h(x, t), & -\infty < x < +\infty \\
  u(0, x) = 0, \ u_t(0, x) = 0.
\end{cases}
\]
Show that if \( h(x, t) = 0 \) inside the right triangle \( Q \),
then \( u(x, t) = 0 \) in \( Q \).