Final Review

1st unit (Practice midterm 1)
- Definitions + Right hand rule
- Compute Dot Products \((\vec{u} \cdot \vec{v})\), cross products \((\vec{u} \times \vec{v})\), Norms \(|\vec{u}||\), \((2x2)(3x3)\) determinants
- Formulas \(\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta\), \(\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}\)
- Area of parallelogram with legs \(\vec{u} + \vec{v}\).
- Volume of parallelepiped by taking the abs. value of the determinant
- Where rows or columns have the vectors of the legs.

\[
\begin{pmatrix}
  u_1 & u_2 & u_3 \\
  v_1 & v_2 & v_3 \\
  w_1 & w_2 & w_3
\end{pmatrix}
\]

- Eqn for a plane, Parametric Eqn for a line, Eqn for a sphere
- Problems worth looking at from practice midterm one:
  - 2-7, 9, 10 (parametric only), 11-17

2nd unit (Practice midterm 2)
- Know what Curvature, tangential/normal acceleration, measure
  - Have some understanding of \(\vec{T}, \vec{N}, \vec{B}\)
  - Curvature: Circle with radius \(R\), \(C = \frac{1}{R}\) (does not depend on speed)
  - Tangential accel: break + gas, normal acceleration = steering wheel
  - \(\vec{T}\) (unit tangent) = direction car is moving, \(\vec{N}\) normal to car
  - \(\vec{B}\) (unit binormal) = directly up or down when tilting on car

- Be able too compute partial derivatives
  - Gradient Vector = (compute) - Direction = Direction set; it increases the fastest, also \(\perp\) to level sets.
  - Directional Derivative: \(\frac{\partial f}{\partial \vec{e}} = \vec{e} \cdot \nabla f\)
  - Chain Rule: \(\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}\)
  - Can be used to find tangent planes, Lagrange multiplier, Magnitude = largest rate of change of \(f\) at a pt.

- Finding tangent planes/linear approximations
- Chain rule
- Max/Min problems
  - Find critical points
  - Find max/min of function \(f(x,y)\) in region \(g(x,y) \leq C\)
  - Lagrange multiplier problem
    - Step 1: Assume \(g \leq 0\) \((f + g \cdot \lambda = \text{const})\)
    - Step 2: Assume \(g \geq 0\), \(f - \lambda g = \text{const}\)
Problems (2nd practice midterm 2)

9-10, 12, 14, 15

3rd unit (practice midterm 3)

- Divergence \( \nabla \cdot \mathbf{F} = \text{Div} \mathbf{F} = \nabla \cdot \mathbf{F} \)
- Curl \( \nabla \times \mathbf{F} = \text{Curl} \mathbf{F} = \nabla \times \mathbf{F} \)

- \( \text{Div} \mathbf{F} \) measures how much the "flow" with velocity field \( \mathbf{F} \) spreads out.
- \( \text{Curl} \mathbf{F} \) measures rotation under that same flow.
  - Direction: axis of rotation (RHS rule = direction of rot.)
  - Magnitude: Angular Velocity

- Theorem: If \( \text{Curl} \mathbf{F} = \nabla \times \mathbf{F} = 0 \) then there exists a potential function \( \Phi \), (i.e. \( \nabla \Phi = \mathbf{F} \))
  - (and you are in an simply connected domain)
  - (\( \mathbf{F} \) is conservative means (there exists) \( \Phi \) such that \( \mathbf{F} = \nabla \Phi \))

- Definition: Irrotational (\( \nabla \times \mathbf{F} = 0 \)), Incompressible (\( \nabla \cdot \mathbf{F} = 0 \))

Integration

- Integral of \( \Phi \) on the set \( \Omega \) = \( \text{size of } \Omega \) \times \( \text{average value of } \Phi \) on \( \Omega \)

- \( \int \mathbf{F} \cdot \mathbf{r} \, ds \)

- \( \text{Mass} = \text{size} \times \text{avg. density} \)
- \( \text{Total charge} = \text{size} \times \text{charge density} \)
- \( \text{Flux} = \int \mathbf{F} \cdot \mathbf{A} \, ds \)

- \( \text{Divergence Theorem:} \)

- \( \int_S (\nabla \cdot \mathbf{F}) \, dA = \int_V \nabla \cdot \mathbf{F} \, dV \)

3rd-4th Units

Fundamental Theorem of Line Integrals (Not on cheat sheet)

- \( \int_C \mathbf{F} \cdot d\mathbf{r} = \Phi (\text{end}) - \Phi (\text{start}) \)

- \( \mathbf{F} \) conservative

- \( \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C (\nabla \Phi) \cdot d\mathbf{r} \)

- \( \int_C \mathbf{F} \cdot d\mathbf{r} = \Phi (\text{end}) - \Phi (\text{start}) \)

- \( \mathbf{F} \) conservative

Stokes' Theorem (on cheat sheet) NOT ON TEST

- \( \int_C (\nabla \times \mathbf{F}) \cdot d\mathbf{r} = \int_S \mathbf{F} \cdot d\mathbf{A} \)

Green's Theorem (on cheat sheet)

- \( \int_C (\partial_y F - \partial_x E) \, dx \, dy = \int_S (\partial_x E + \partial_y F) \, dx \, dy \)

Recognize (on final)

\( \int \mathbf{F} \cdot d\mathbf{r} = \int_C (\nabla \Phi) \cdot d\mathbf{r} \)

\( \int_S \nabla \Phi \cdot d\mathbf{A} = \int_V \nabla \cdot \mathbf{F} \, dV \)

\( \int_C (\nabla \times \mathbf{F}) \cdot d\mathbf{r} = \int_S \mathbf{F} \cdot d\mathbf{A} \)

\( \int_S (\partial_y F - \partial_x E) \, dx \, dy = \int_C (\partial_x E + \partial_y F) \, dx \, dy \)
\[ \sum_{\mathcal{C}} S_{\mathcal{C}} = \int_{a}^{b} \sum_{\mathcal{C}} F(x) \, dF(x) \, dF(x) \]
\[ \sum_{\mathcal{C}} F(x) \, dx = \int_{a}^{b} F(x) \cdot F'(x) \, dx \]
\[ \sum_{\mathcal{C}} \sum_{\mathcal{C}} S_{\mathcal{C}}(x) \cdot \sum_{\mathcal{C}} N(x) \, dA \]
\[ \sum_{\mathcal{C}} \sum_{\mathcal{C}} F(x) \cdot N(x) \, dA \]

- **Difficulties when computing integrals?**
  - Integration by parts, u, v, etc. (rule II shift)
  - Orders of Integration
    - Bound of Integration
  - Choice of coordinate system (Spherical, Cylindrical, Polar, Cartesian)
  - Making sure orientation is correct (SA integrations)
  - Parameterizations of curves + surfaces (Sphere, cone, plane, paraboloid, cylinder, tetrahedron)

- Practice midterm problems: All
- Practice final problems: All

Integrating 4th - A.N. (best surface for data from)