Course Description

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Prerequisite: a course in complex analysis.

Familiarity with Hilbert spaces is helpful, but not required. No need for previous background in general functional analysis and Banach spaces.

Purpose: Introduce the students to some modern viewpoints toward classical analysis. The emphasis will be interaction between function theory and operator theory, together with an algebraic framework in terms of module theory.

Note: This course is not intended to teach general functional analysis or operator theory. Instead, my hope is that students can appreciate how to use functional analysis before taking up the burden of learning it systematically.

Sample problems: With a minimal background in complex analysis, one can easily understand and try to work on the following two problems:

1. Pick \( n = 2006 \) distinct points in the unit disc
   \[ z_i \in \mathbb{D}, \]
   and prescribe arbitrarily \( n = 2006 \) complex numbers
   \[ w_i \in \mathbb{C}, \]
   when can one find a bounded analytic function on the disc, with sup norm \( \leq 1 \), such that
   \[ f(z_i) = w_i? \]

2. Two ways to define a (pseudo-) distance on a domain \( U \subset \mathbb{C}^n \) are through the Caratheodory distance, and the Kobayashi distance. Prove that if \( U \) is a bounded convex domain, then the two distances agree.
The above two problems, which make no mentioning of Hilbert spaces, can be solved with roughly the same set of techniques. That is, the **function theory and operator theory centering around the Hardy space over the unit disc** $H^2(\mathbb{D})$.

Along the way, we have the following

**Topics to be studied:**

- More on bounded analytic functions on the disc $H^\infty(\mathbb{D})$;
- More on the Hardy space $H^2(\mathbb{D})$;
- Beurling’s Theorem, and its applications
- reproducing kernels on function spaces
- a mini-course on Hilbert spaces, with a little on weak topologies
- basics for contraction operators
- Sz. Nagy -Foias’ Dilation Theorem
- Commutant Lifting Theorem
- von Neumann’s inequality
- Hilbert modules, with algebraic viewpoints

**Grading:** based on homework and attendance. No exams.

**Text:** No text, but I will type up notes for each class!!!