TRIGONOMETRY

MATH 150 - Practice Exam 2
Fall Semester, 2009

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Show all your work in the space provided under each question. Please write neatly and present your answers in an organized way. You may use your notecard, but calculators are not allowed.

(12 pts) 1. Verify the identity: \( \cot^2 \alpha - \cos^2 \alpha = \cot^2 \alpha \cos^2 \alpha \).

\[
\cot^2 \alpha - \cos^2 \alpha = \frac{\cos^2 \alpha}{\sin^2 \alpha} - \cos^2 \alpha \\
= \frac{\cos^2 \alpha - \sin^2 \alpha \cos^2 \alpha}{\sin^2 \alpha} \\
= \frac{\cos^2 \alpha (1 - \sin^2 \alpha)}{\sin^2 \alpha} \\
= \frac{\cos^2 \alpha (\cos^2 \alpha)}{\sin^2 \alpha} \\
= \cot^2 \alpha \cos^2 \alpha
\]
(6 pts) 2. Find all solutions of the equation \( \cos \beta = -1 \).

\[ \theta = \pi + 2n\pi \quad \text{where } n \text{ is an integer} \]

(6 pts) 3. Find all solutions of the equation \( 2\cos^2 u + 3\cos u = 2 \) that are in the interval \([0, 2\pi]\).

\[
2\cos^2 u + 3\cos u - 2 = 0
\]

\[
(2\cos u - 1)(\cos u + 2) = 0
\]

\[
2\cos u - 1 = 0 \quad \text{or} \quad \cos u + 2 = 0
\]

\[
\cos u = \frac{1}{2} \quad \text{or} \quad \cos u = -2
\]

\[
\text{no solutions}
\]

\[
\cos u = \frac{1}{2} \quad \text{or} \quad \cos u = -2
\]

\[
\frac{\pi}{3}, \frac{5\pi}{3}
\]
Suppose $\alpha$ and $\beta$ are second quadrant angles with $\sin \alpha = \frac{4}{5}$ and $\cos \beta = -\frac{12}{13}$. Find:

(6 pts) 6. $\sin(\alpha + \beta)$

First find $\sin \beta$ , $\cos \alpha$.

\[
\cos^2 \beta + \sin^2 \beta = 1
\]
\[
\frac{144}{169} + \sin^2 \beta = 1
\]
\[
\sin^2 \beta = \frac{25}{169}
\]
\[
\sin \beta = \frac{5}{13}
\]

So

\[
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta
\]
\[
= \frac{4}{5} \cdot \left(-\frac{12}{13}\right) + \frac{3}{5} \cdot \left(\frac{5}{13}\right)
\]
\[
= -\frac{48}{65} - \frac{15}{65}
\]
\[
= -\frac{63}{65}
\]

(6 pts) 7. $\cos(\alpha + \beta)$

\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
\]
\[
= -\frac{3}{5} \cdot \left(-\frac{12}{13}\right) - \frac{4}{5} \cdot \frac{5}{13}
\]
\[
= \frac{16}{65}
\]
(6 pts) 4. Express as a cofunction of a complementary angle: \( \sin 34^\circ \).

\[
\sin 34^\circ = \cos (90^\circ - 34^\circ) = \cos 56^\circ
\]

(6 pts) 5. Find the exact value of \( \cos \left( \frac{7\pi}{12} \right) \). (Leave your answer involving integers and square roots of integers.)

\[
\cos \frac{7\pi}{12} = \cos \left( \frac{3\pi}{4} + \frac{\pi}{3} \right) = \cos \left( \frac{\pi}{4} + \frac{\pi}{3} \right)
\]

\[
= \cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3}
\]

\[
= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}
\]

\[
= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}
\]

\[
= \frac{1 - \sqrt{3}}{2\sqrt{2}}
\]
(6 pts) 8. Suppose \( \cos \theta = \frac{1}{5} \), \( 0 < \theta < 90^\circ \). Find the exact value of \( \cos 2\theta \).

\[
\cos^2 \theta + \sin^2 \theta = 1 \\
\frac{1}{25} + \sin^2 \theta = 1 \\
\sin^2 \theta = \frac{24}{25} \\
\sin \theta = \frac{\sqrt{24}}{25} \\

\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \frac{1}{25} - \frac{24}{25} = -\frac{23}{25}
\]

(6 pts) 9. Suppose \( \sin \theta = -\frac{3}{5} \), where \( -\frac{\pi}{2} < \theta < 0 \). Find the exact value of \( \sin 2\theta \).

\[
\sin^2 \theta + \cos^2 \theta = 1 \\
\frac{9}{25} + \cos^2 \theta = 1 \\
\cos^2 \theta = \frac{16}{25} \\
\cos \theta = \frac{4}{5} \\

\sin 2\theta = 2 \sin \theta \cos \theta = 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = -\frac{24}{25}
\]
(6 pts) 10. Express as a product: \( \cos 3x + \cos 5x \).

\[
\cos 3x + \cos 5x = 2 \cos \left( \frac{3x + 5x}{2} \right) \cos \left( \frac{3x - 5x}{2} \right)
\]

\[
= 2 \cos 4x \cos(-x)
\]

\[
= 2 \cos 4x \cos x
\]

(6 pts) 11. Find all solutions in the interval \([0, 2\pi]\) to the equation:

\[
\sin 4x + \sin 2x = 0
\]

\[
2 \cos 2x \sin 2x + \sin 2x = 0
\]

\[
(2 \cos 2x + 1) \sin 2x = 0
\]

\[
2 \cos 2x + 1 = 0 \quad \text{or} \quad \sin 2x = 0
\]

\[
2 \cos 2x = -1
\]

\[
\cos 2x = -\frac{1}{2}
\]

\[
x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}
\]

\[
2x = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots
\]

\[
x = 0, \pi, 2\pi, 3\pi, 2\pi, \ldots
\]

in \([0, 2\pi]\) solutions are \(0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \ldots\)

Note: This can also be solved using a product-to-sum formula.
(6 pts) 12. Verify the identity:

\[ \csc 2x = \frac{1}{2} \csc x \sec x \]

\[ \frac{1}{\sin 2x} = \frac{1}{2 \sin x \cos x} = \frac{1}{2} \frac{1}{\sin x} \cdot \frac{1}{\cos x} \]

\[ = \frac{1}{2} \csc x \sec x \]

(6 pts) 13. The expected low temperature in Fairbanks, Alaska may be approximated by

\[ T = 36 \sin \left( \frac{2\pi}{365} (t - 101) \right) + 14. \]

How many days during the year is the low temperature expected to be above \(-4^\circ\)?

\[ 36 \sin \left( \frac{2\pi}{365} (t - 101) \right) + 14 > -4 \]

\[ \sin \left( \frac{2\pi}{365} (t - 101) \right) > \frac{1}{3} \]

\[ \frac{\pi}{6} \leq \frac{2\pi}{365} (t - 101) \leq \frac{5\pi}{6} \]

\[ \frac{365}{12} \leq t - 101 \leq \frac{365 \cdot 5}{12} \]

\[ \text{Days} = \frac{365.5}{12} - \frac{365}{12} = \frac{365 \cdot 1}{3} \approx 122 \]
(8 pts) 14. At a certain distance from a tower, the angle of elevation to the top is 30°. 100m closer, the angle of elevation is 45°. How tall is the tower?

\[ x = \text{height of tower} \]

\[ \frac{x}{100+d} = \tan 30° = \frac{\frac{1}{2}}{\sqrt{3}} \]

\[ x = \frac{1}{\sqrt{3}} (100 + d) \]

\[ \frac{x}{d} = \tan 45° = 1 \]

So \[ x = \frac{1}{\sqrt{3}} (100 + x) \]

\[ x \left(1 - \frac{1}{\sqrt{3}}\right) = \frac{100}{\sqrt{3}} \]

\[ x \left(\frac{\sqrt{3} - 1}{\sqrt{3}}\right) = \frac{100}{\sqrt{3}} \]

\[ x = \frac{100}{\sqrt{3} - 1} \]

(8 pts) 15. A ship leaves port at noon sailing in the direction S 27° W at 12 miles per hour. Another ship leaves the same port at 1 pm, sailing in the direction S 63° E at 10 miles per hour. How far apart are the ships at 2 pm?

\[ x^2 = 10^2 + 24^2 \]

\[ x^2 = 100 + 576 = 676 \]

\[ x = 26 \text{ miles} \]