1. Let \( W_1 \subseteq V \) and \( W_2 \subseteq V \) be two subspaces of a vector space \( V \). Assume that \( V \) is equal to the union \( V = W_1 \cup W_2 \). Prove that either \( V = W_1 \) or \( V = W_2 \).
2. Let $U \subseteq V$ be a subspace of a finite dimensional vector space $V$. Prove that there exists a subspace $W \subseteq V$ such that $V$ is the internal direct sum $V = U \oplus W$. 
3. Let $V$ be a vector space over a field $F$. Let $f, g : V \rightarrow F$ be two linear functionals such that $f(v) = 0$ implies $g(v) = 0$. Prove there exists a scalar $\lambda \in F$ such that $g(v) = \lambda \cdot f(v)$ for all $v \in V$. 
4. Let $F \subseteq K$ be a field extension, and assume $\alpha \in K$ is an algebraic element of odd degree. Prove that $\alpha^2$ is an algebraic element of odd degree, and that $F(\alpha^2) = F(\alpha)$. 
5. Let $f, g \in F[x]$ be two polynomials with coefficients in a field $F$. Let $h_1 \in F[x]$ be a greatest common divisor of $f$ and $g$ in $F[x]$. Let $F \subseteq K$ be a field extension. Let $h_2 \in K[x]$ be a greatest common divisor of $f$ and $g$ in $K[x]$. Prove that $h_1$ and $h_2$ are associates in $K[x]$. (Hint: In a Euclidean domain $R$, a GCD is a generator of the principal ideal consisting of all linear combinations.)