Calculus is the branch of mathematics which studies quantities undergoing change. Calculus is used to study the change in the position of planets with respect to time or the change in demand for gas guzzling cars with respect to the price of gasoline. Since almost everything in the world changes, calculus has applications in every part of science and engineering. Yet, in its narrowest sense, calculus may be regarded as treating two geometric problems: computing the tangent lines to the graphs of functions and computing the area of regions bounded by the graphs of functions.

Calculus problems are more difficult than problems from algebra or trigonometry. Most students need to go over the more difficult problems several times. One technique many students use to cope with this is to take careful class notes. The act of getting the ideas and methods outlined in the lecture down clearly on paper is a powerful aid to memory. (It doesn't matter if the ideas are already in the book. Taking them down in writing helps fix them in your memory.) It is also a good idea to review your notes repeatedly, partly to identify areas of confusion, partly to review.


The Larson, Hostetler & Edwards text is available at Varney's and the K-State Union Bookstore, either hardback or paperback (ch.1-6).

**PROCEDURE:** This course is organized along the lecture-recitation method. Each week you are expected to attend two lectures and two recitation classes. The material to be covered in each lecture along with the corresponding exercise set is indicated on the attached assignment sheet. You should read the assigned material before each lecture and take careful class notes during lecture. After each lecture, reread the material, review your class notes and do as many of the assigned exercises as you can before the next recitation class. Try to get the remaining exercises worked in your recitation class, but one way or another, you should eventually have all of the assigned exercises solved. Problem solving is the most important aspect of this course. Most students also find that they frequently need one-on-one instruction to fully master the techniques of this course. Experience has shown that the students who come to the office hours of the instructor regularly, or go to the help room sessions tend to do better in the course.

**GRADING:** Your recitation instructor will administer your exams and determine your final letter grade. You may earn 720 points in this course: 100 points on each of the three hour exams, 200 points on the final exam, 120 points on the homework, and 100 points in recitation. Your recitation instructor will explain exactly how these 100 points will be awarded but it will be on the basis of attendance, class participation, and quizzes. Homework is due weekly, to be turned in before 5:00 p.m. to the homework boxes each Monday (except for the first assignment, due Tuesday, Sept. 3, and the 14th assignment, due Tuesday, Dec.3). Included with this syllabus are an Algebra Worksheet and Projects 1, 2, and 3. These are part of the homework assignments 1, 4, 8, and 12, and should be handed in along with the problems assigned from the text. Letter grades will be assigned for each exam, but these should only be considered an indication of your progress. Your final letter grade will be determined from your total accumulated points, including the points you earn in recitation class.

**EXAMINATIONS:** Hour exams will be held on September 17, October 15, and November 12 from 7:30 p.m. to 8:30 p.m. The final exam will be held on Wednesday, December 18 from 7:00 to 9:00 p.m. Room assignments for the hour exams will be announced during the second week of classes, and room assignments for the final exam will be announced during the last week of classes. Room assignments for the exams are by recitation section. If you are enrolled also in a course whose final exam conflicts, you will need to arrange to take an alternate final examination in the other course. A basic requirement of this course is that you are able to take the exams as scheduled. If you expect to miss an hour exam and have a reasonable excuse (for example, illness or University business), notify your recitation instructor as far in advance of the exam as possible. In this case, your final letter grade will be determined by your other work in the course. If you miss an hour exam without the consent of your recitation instructor, your grade on that exam will be zero. Should you miss either two hour exams or the final for an excused reason, you will receive a grade of I, incomplete. There are no "make-up" exams.

Most exam questions will be modifications of homework problems or examples from the text or lectures. You should bring sharpened soft lead pencils and your KSU identification card to each exam. You may use a calculator on the exams but no written notes or other material. Partial credit will be assigned to your solutions when deserved, but this is completely determined by the grader. If you feel that one of your solutions has been mis-graded, seek clarification from the grader within seven days. A worked out copy of each exam will be posted on the Calculus I bulletin board in the hallway outside Cardwell 128 at the conclusion of the exam. You should study it. Your hour exam papers, each marked with a numerical grade, will be returned to you in recitation class. Your final exam paper will be kept for at least one year by your recitation instructor. Copies of old exams are at the Reserve Desk in Hale Library and review problems are included with this syllabus.
GENERAL INFORMATION: Information concerning the location of exams, solutions to exams, grading scales on exams, help session locations and schedules, and other information relevant to the course will be posted on the Calculus I bulletin board in the hall outside Cardwell 128. Your recitation instructor will announce office hours during which you may seek help. In addition, help sessions are held Monday through Thursday during the day. A help session schedule will be posted on the main bulletin board across from the Mathematics Office (CW 138). Several instructors will be present to help you. Tutors for Calculus can be located through the Mathematics Department or through numerous service organizations on campus. Free tutoring in small groups is available in Leasure 201.

The most common reasons for difficulty with Calculus are poor algebra skills and failure to study consistently. Calculus is an acquired skill. Developing it takes practice. Attend all recitations and lectures. Read your book and class notes carefully and repeatedly. Visit with your recitation instructor during office hours; we will help you clear up problems.

If you have any condition, such as a physical or learning disability, which will make it difficult for you to carry out the work as I have outlined it or which will require academic accommodations, please notify me in the first two weeks of classes.

ACADEMIC DISHONESTY: Plagiarism and cheating are serious offenses and may be punished by failure on the exam, paper or project, failure in the course and/or expulsion from the University and a letter placed in your permanent file. For more information refer to the academic dishonesty policy in the University handbook.

Dr. John Maginnis
114 Cardwell Hall
Coordinator

HOUR EXAMINATIONS: September 17, October 15, and November 12 (7:30 to 8:30 p.m.)
FINAL EXAMINATION: December 18 (7:00 to 9:00 p.m.)
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### Homework Problems - Due Mondays before 5:00 p.m.

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### Review Problems (do not hand in)

- p.36  11,14,30,31,32,47
- p.38  1,4,5,7,8,14
- p.88  3,5,13,15,17,20,21,31,34,35,49,51,57,62,65,70
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- p.310 3,16
- p.308 50,55,57,59,61,85
- p.310 6,8
- p.476 1,4,5,6,9,15,22,29,30,33,34,39,40,41,43
- p.478 5,13,16
Algebra Worksheet – part of Homework #1 - due Tuesday, September 3

1. Factor.
   (a) $2x^5 + x^4 - 6x^3$  
   (b) $12x^3y^2 - 3xy^4$

2. Simplify.
   (a) $\frac{2}{x} - \frac{1}{x+1}$
   (b) $(3^0 - 2^{-3})^{-2}$
   (c) $\frac{(x^2y^3)^4(xy^4)^3}{(x^3y^2)^0(x^5y)^2}$

3. Expand $(x - 2)^3$.

4. Find the least common denominator, and subtract the fractions.
   \[ \frac{x + 2}{x^2(x - 1)(x + 1)} - \frac{2x + 1}{x(x + 1)^2} \]

5. Solve:
   (a) $x^2 = 9$  
   (b) $x^2 = x + 2$
   (c) $x^2 > 9$  
   (d) $x^2 \leq x + 2$

6. (a) Use the quadratic formula to solve $x^2 - 6x + 3 = 0$.
   (b) Give a factorization (over the real numbers) of $x^2 - 6x + 3$.

7. Let $y = -x^2 + 4x - 3$.
   (a) Find the $x$- and $y$-intercepts.
   (b) Find the $x$- and $y$-coordinates of the vertex of the parabola.
   (c) Solve the equation $y \geq 0$.
   (d) Graph the parabola.
8. If \( \tan(\theta) = \frac{3}{4} \) and \( \sin(\theta) < 0 \), find \( \cos(\theta) \).

9. Find every angle \( \theta \) with \( 0 \leq \theta \leq 2\pi \) radians, and \( 2\sin^2(\theta) + \cos(\theta) = 2 \).

10. Solve for \( x \) if \( y = \frac{2x}{x - 3} \).

11. Solve for \( r \) if \( A = P \left(1 + \frac{r}{n}\right)^{nt} \).

12. Give the vertical and horizontal asymptotes of \( y = \frac{3x^4}{2x^4 - 16x} \).

13. Use long division to rewrite \( \frac{4x^3 - 3x - 1}{2x^2 + 3x + 2} \).

14. A conical funnel has a radius of 3 inches and a height of 6 inches. Oil fills the funnel to a height of 2 inches. Find the volume of the oil.

15. A college is building a track in the shape of a rectangle with a semicircle at each end. The perimeter is \( 200\pi \) feet and the area of the rectangular region is \( 5000\pi \) square feet. Find the length and width of the rectangular region.
Project 1 - part of HW 4 - due Monday, September 23

P1-1. (a) Joey records the temperature outside his home every three hours for two days. On a graph with time on the horizontal axis and temperature on the vertical axis, plot 17 points that might represent Joey’s records from six o’clock Friday evening to six o’clock Sunday evening. What type of function might be used to describe a curve fitting this data?

(b) When a cake is removed from a 350° oven, it initially cools quickly but then more gradually approaches the room temperature of 70°. Sketch a graph that might represent the temperature of the cake as a function of time. What type of function might be used to describe this curve?

P1-2. A heavily loaded truck travels 120 miles from Manhattan to Kansas City at 60 miles per hour. The driver quickly detaches his load, and immediately returns to Manhattan in his cab at 80 mph.

(a) Sketch a graph with time on the horizontal axis and distance from Manhattan on the vertical axis (from zero to 120 miles).

(b) Sketch a graph with time on the horizontal axis and the total distance travelled on the vertical axis (from zero to 240 miles).

(c) Compute the average speed for the round trip. Is your answer greater than or less than 70 mph? Give a geometric interpretation of this average speed using your graph in part (b).

P1-3. Let \( f(x) = \frac{7 - \sqrt{x + 40}}{9 - x} \).

(a) Use a calculator to evaluate this function at the following \( x \)-values: 8.9, 8.99, 8.999, 9.1, 9.01, 9.001. Round your answers to seven decimal places.

(b) Evaluate \( \lim_{x \to 9} f(x) \) algebraically by multiplying the numerator and denominator by \( 7 + \sqrt{x + 40} \), and simplifying.
P1-4. Let \( g(x) = \sin \left( \frac{\pi}{x} \right) \), where \( \pi = 3.1415926535 \).

(a) Evaluate this function at the \( x \)-values .5, .1, .01, .001, and .0001.

(b) Evaluate this function at the \( x \)-values .4, .08, .016, .0032, and .00064.

(c) Use a graphing calculator or computer to graph \( y = \sin \left( \frac{\pi}{x} \right) \) with the window \(-1 \leq x \leq 1\) and \(-1 \leq y \leq 1\).

Note: your answers should illustrate that plugging in numbers to estimate limits may be misleading, and also give evidence that \( \lim_{x \to 0} \sin\left( \frac{\pi}{x} \right) \) does not exist.

P1-5. Use the graph of \( y = f(x) \) below to estimate the following.

(a) \( f(0) \) \hspace{1cm} (b) \( f(1) \) \hspace{1cm} (c) \( f(-1) \)

(d) \( \lim_{x \to 0} f(x) \) \hspace{1cm} (e) \( \lim_{x \to 1} f(x) \) \hspace{1cm} (f) \( \lim_{x \to -1^+} f(x) \)

(g) \( \lim_{x \to -1^-} f(x) \) \hspace{1cm} (h) \( \lim_{x \to +\infty} f(x) \) \hspace{1cm} (i) \( \lim_{x \to -\infty} f(x) \)

(j) \( \lim_{h \to 0^+} \frac{f(h) - f(0)}{h} \) \hspace{1cm} (k) \( \lim_{h \to 0^-} \frac{f(h) - f(0)}{h} \)

(l) Is \( y = f(x) \) continuous at \( x = 0 \)?

(m) Is \( y = f(x) \) continuous at \( x = 1 \)?

(n) Is \( y = f(x) \) differentiable at \( x = 0 \)?
P1-6. Denote \( E_1 = \frac{1}{2}mv^2 \) and \( E_2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 \).

(a) Compute \( \lim_{v \to c^-} E_1 \) and \( \lim_{v \to c^-} E_2 \).

(b) Compute \( \lim_{c \to \infty} E_2 \).

(Note: \( E_1 \) is the kinetic energy of an object of mass \( m \) moving with velocity \( v \) in Newtonian physics, and \( E_2 \) is the relativistic kinetic energy of an object of rest mass \( m \) moving with velocity \( v \). Here \( c \) denotes the speed of light, and part (b) can be interpreted as comparing the formulas for small velocities. In part (a) we take left-hand limits since objects cannot move faster than light.)
P2-1.  (a) If a line $L_1$ meet a horizontal line $L_2$ at an angle $\theta_2$ (measured in a counterclockwise direction from $L_2$ to $L_1$), show that the slope of $L_1$ equals $\tan(\theta_2)$. (Try two cases depending on whether $\theta_2$ is acute or obtuse.)

(b) If a line $L_1$ meets a vertical line $L_3$ at an angle $\theta_3$ (measured in a clockwise direction from $L_3$ to $L_1$), show that the slope of $L_1$ equals $\cot(\theta_3)$.

(c) This problem is related to the property of a parabolic mirror reflecting light to a focal point. Consider the parabola $y = x^2$ and a beam of light traveling downward (parallel to the y-axis) which meets $y = x^2$ at a point $(a, a^2)$. The light will reflect so that the angle of incidence $\theta$ with the tangent line to $y = x^2$ at $x = a$ equals the angle of reflection. Show the slope of the tangent line equals $\cot(\theta)$ and the slope of the reflected light equals $\cot(2\theta)$. Use calculus to express the slope of the tangent line in terms of $a$, and use the trigonometric identity $\cot(2\theta) = \frac{\cot^2(\theta) - 1}{2\cot(\theta)}$ to express the slope of the reflected light in terms of $a$. Finally, show the reflected light passes through the focal point $(0, \frac{1}{4})$. 

\[
\begin{align*}
\text{\theta} & \quad \text{\theta} \\
y &= x^2 & (a, a^2) \\
\end{align*}
\]
P2-2. For the function \( f(x) = x^3 \), compute the limits:

a) \[
\lim_{h \to 0} \frac{f(x + h) - f(x - h)}{2h}
\]

b) \[
\lim_{h \to 0} \frac{f(x + h) - 2f(x) + f(x - h)}{h^2}
\]

Try to explain why these limits yield \( f'(x) \) and \( f''(x) \).

P2-3. The graph of \( y = g(x) \) is given below.

a) Sketch the graph of the derivative \( g'(x) \).

b) Sketch the graph of the second derivative \( g''(x) \).

P2-4. A large vase, 24 inches tall, is pictured here. Water is poured into the vase at a constant rate (so \( \frac{dV}{dt} \) is constant). Sketch a graph of the height of the water as a function of time. Describe the features of your graph that occur at the heights of 6 inches and 18 inches, the widest and most narrow parts of the vase.
P2-5. Let \( y = f(x) = \frac{x^3 - 6x^2 + 4x + 8}{x^2 - 4x - 8} \).

a) Give the \( y \)-intercept.

b) Use the quadratic formula to find the vertical asymptotes, then round your answers to one decimal place.

c) There are three \( x \)-intercepts; one is an integer. First find the integer \( x \)-intercept, then factor, and use the quadratic formula to find the other two \( x \)-intercepts. Round your answers to one decimal place.

d) Rewrite the function \( f(x) \) using long division. Then give the line, in the form \( y = mx + b \), which is a slant asymptote of the function \( f(x) \).

e) Show the derivative \( f'(x) = \frac{x^4 - 8x^3 - 4x^2 + 80x}{(x^2 - 4x - 8)^2} \).

f) Find the \( x \)-coordinates of all critical points. Two are integers; use the quadratic formula to find the other two, and round to one decimal place. Also give the \( y \)-coordinates of these four critical points.

g) Give the intervals (\( x \)-coordinates) where \( f(x) \) is an increasing function.

h) Show the second derivative \( f''(x) = \frac{8(x^3 - 6x^2 + 48x - 80)}{(x^2 - 4x - 8)^3} \).

i) Find the intervals (\( x \)-coordinates) where \( f(x) \) is concave up.

j) Give the \( x \)- and \( y \)-coordinates of all inflection points.

k) Sketch a graph of \( y = f(x) \).
P2-6. A lighthouse stands 1000 feet from a very long straight cliff on the coastline. The light rotates at a constant rate, making one full revolution every 20 seconds. Let \( \theta \) denote the angle between the beam of light and the line forming the 1000 foot distance from the lighthouse to the cliff.

a) Find \( \frac{d\theta}{dt} \) in radians per second.

b) Find the speed with which the spot of light moves along the cliff at the instant when \( \theta = \frac{\pi}{4} \) radians.

c) Find the speed with which the spot of light moves along the cliff at the various instants when \( \theta = 1.5, 1.57, \) and \( 1.57079 \) radians (all slightly less than \( \theta = \frac{\pi}{2} \), when the beam of light is parallel to the cliff). Do you believe these answers? Note the speed of light is approximately 983 million feet per second.
**Project 3 - part of HW 12 - due Monday, November 18**

**P3-1.** Let \( f(x) = 8 \cdot \sqrt{x+1} \). Note that \( f(3) = 16 \), and so we have a very rough approximation \( f(x) \approx 16 \) near \( x = 3 \), and so \( 8\sqrt{x+1} - 16 \approx 0 \) near \( x = 3 \).

a) Show that \( \lim_{x \to 3} \frac{8\sqrt{x+1} - 16}{x - 3} = 2 \) by multiplying the numerator and denominator by \( 8\sqrt{x+1} + 16 \).

Thus near \( x = 3 \) we have \( 8\sqrt{x+1} - 16 \approx 2(x-3) \) and so \( f(x) \approx 2x+10 \).

b) Find the derivative \( f'(x) \) and give the equation of the tangent line at \( x = 3 \), in the form \( y = mx + b \).

c) Note that \( 8\sqrt{x+1} - 2x - 10 \approx 0 \) near \( x = 3 \). Show that
\[
\lim_{x \to 3} \frac{8\sqrt{x+1} - 2x - 10}{(x - 3)^2} = \frac{-1}{8}
\]
by multiplying the numerator and denominator by \( 8\sqrt{x+1} + 2x + 10 \).

Thus near \( x = 3 \) we have
\[
8\sqrt{x+1} - 2x - 10 \approx \frac{-1}{8} (x - 3)^2,
\]
yielding a quadratic approximation
\[
f(x) \approx \frac{-1}{8} x^2 + \frac{11}{4} x + \frac{71}{8}.
\]

d) Denote \( g(x) = \frac{-1}{8} x^2 + \frac{11}{4} x + \frac{71}{8} \).

Compute \( f''(x) \), and show that \( f(3) = g(3) \), \( f'(3) = g'(3) \), and \( f''(3) = g''(3) \).
P3-2. Let \( y = f(x) = -9x^3 + 40x^2 - 33x + 1 \).

a) Compute the \( y \)-coordinates corresponding to the \( x \)-coordinates \( x = 0, 1, 2, 3, \) and 4. Explain why your answers show that \( y = f(x) \) has exactly three \( x \)-intercepts.

b) One method for estimating the \( x \)-intercepts is to iteratively divide intervals in half. Compute \( f(3.5) \), showing that one \( x \)-intercept lies between \( x = 3 \) and \( x = 3.5 \). Continue, computing \( y \)-coordinates corresponding to \( x = 3.25, 3.375, 3.3125, 3.34375, 3.359375, \) and 3.3671875.

c) Use Newton’s Method \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \) to estimate an \( x \)-intercept, starting with \( x_0 = 0 \). You should find that \( x_3 \) yields an excellent approximation to one \( x \)-intercept.

d) Use Newton’s Method again, but starting with \( x_0 = \frac{1}{2} \) (note this is the value obtained by interpolation, using a straight line between the points with \( x = 0 \) and \( x = 1 \)). The method now takes much longer, and approaches a different \( x \)-intercept than the one you found in part (c). Give a final answer accurate to four decimal places.

e) Find the third \( x \)-intercept, accurate to four decimal places, using any method.
P3-3. The passenger in a car driving on a straight road records the velocity every two seconds by reading the speedometer. Later, he changes the numbers into units of feet per second, yielding the following chart:

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity</td>
<td>50</td>
<td>52</td>
<td>55</td>
<td>59</td>
<td>64</td>
<td>70</td>
</tr>
</tbody>
</table>

a) Approximate the acceleration at the instant \( t = 5 \) seconds by computing the average acceleration from \( t = 4 \) to \( t = 6 \).

b) Approximate the velocity at the instant \( t = 5 \) seconds using interpolation (a straight line between the two points \((4, 55)\) and \((6, 59)\)). Note the acceleration of the car is not constant; do you think the actual velocity of the car at \( t = 5 \) was less than or greater than this interpolated value?

c) Approximate the distance that the car moves during these ten seconds using “left-hand endpoints” \( 0 \leq t \leq 8 \) (the underestimate).

d) Approximate the distance that the car moves during these ten seconds using “right-hand endpoints” \( 2 \leq t \leq 10 \) (the overestimate).
P3-4. A model rocket is shot into the air, a parachute opens, and the rocket lands gently on a nearby hill. The following graph represents the velocity, in the vertical direction, of the rocket as a function of time. The time is measured in seconds and the velocity is measured in feet per second.

a) At what time does the rocket reach its maximal height?

b) At what time does the fuel run out?

c) At what time does the parachute open?

d) The point on the hill where the rocket lands is how many feet above the initial take-off point?
P3-5. Define a function using the integral \( f(x) = \int_1^x \frac{1}{t} \, dt \). (Do not rewrite in terms of logarithms.)

a) Compute \( f'(x) \).

b) Compute \( f(1) \).

c) Let \( a \) be a constant positive number. Make the substitution \( u = \frac{t}{a} \) to rewrite \( f(x) \) as an integral with respect to \( u \). Remember to substitute for the limits of integration.

d) Note it is also true that \( f(x) = \int_1^x \frac{1}{u} \, du \). Show that \( f \left( \frac{1}{a} \right) = -f(a) \).

e) Let \( b \) be a constant positive number. Again using \( u = \frac{t}{a} \), show that \( f(ab) = f(b) - f \left( \frac{1}{a} \right) \). Then combine with part (d) to show that \( f(ab) = f(a) + f(b) \). Also show that \( f \left( \frac{a}{b} \right) = f(a) - f(b) \).