1. Find the inverse of the matrix \( \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix} \) by row reducing an augmented matrix.
2. Find the determinant of the matrix.

\[
\begin{pmatrix}
1 & 0 & 0 & 2 \\
2 & 0 & 1 & 0 \\
0 & -1 & 0 & 1 \\
0 & 2 & -1 & 0 \\
\end{pmatrix}
\]
3. Write the matrix \( \begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix} \) as a product of elementary matrices.
4. Let $A \in M_{n \times n}(F)$ and $B \in M_{n \times n}(F)$ be two square $n \times n$ matrices. Assume that the product $AB$ is invertible. Prove that both $A$ and $B$ are invertible.
5. Let: $T : V \to W$ be a linear transformation of vector spaces over a field $F$. Assume that both $V$ and $W$ are finite dimensional. Let $B_1$ and $B_2$ be two bases for $V$, and let $B_3$ and $B_4$ be two bases for $W$. Let $M = [T]_{B_3}^{B_1}$ and $N = [T]_{B_4}^{B_2}$ be the matrices of the linear transformation $T$ with respect to the various bases. Prove that $\text{rank}(M) = \text{rank}(N)$. 
6. Let $V$ be a finite dimensional vector space over a field $F$. Let $f : V \to F$ and $g : V \to F$ be two linear functionals. Assume the nullspaces satisfy $N(f) \subseteq N(g)$. Prove that there exists a scalar $c \in F$ such that $g(\bar{v}) = cf(\bar{v})$ for all $\bar{v} \in V$ (so that the function $g = cf$).