1. Let $V$ be a vector space over a field $F$ and let $A = \{v_1, v_2, \ldots, v_n\}$ be a subset of $V$ containing $n$ distinct vectors. What does it mean to say

(a) $A$ is linearly independent,

(b) the span of $A$ is $V$,

(c) $A$ is a basis for $V$?

2. Find a basis for the subspace $M$ of $\mathbb{R}^4$ consisting of all solutions $x = \langle x_1, x_2, x_3, x_4 \rangle$ of the system

$$
\begin{align*}
2x_1 - x_2 + x_3 - 2x_4 &= 0 \\
x_1 - x_2 - x_4 &= 0 \\
-x_1 + 4x_2 + 3x_3 + x_4 &= 0.
\end{align*}
$$

Explain why it is a basis.
3. Use elementary row transformations to find the inverse of the matrix

\[
A = \begin{bmatrix}
1 & 0 & 2 \\
-1 & 4 & 1 \\
0 & 1 & 1
\end{bmatrix}.
\]

4. Define \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) by

\[
T(\langle x_1, x_2 \rangle) = \langle 2x_1 - x_2, x_1 + x_2 \rangle.
\]

Find the matrix of \( T \) with respect to the (ordered) basis \( \{v_1, v_2\} \) for \( \mathbb{R}^2 \) where \( v_1 = \langle 1, 1 \rangle \) and \( v_2 = \langle 3, 2 \rangle \).