1. (a) How many 4 letter words are there with no repeated letters?

(b) How many sets of 4 distinct letters can be chosen from the alphabet?

2. Suppose that the numbers 1 through 10 are randomly positioned around a circle. Show that the sum of some set of 3 consecutive numbers must be at least 17.
3. In (a) and (b) determine if the graphs are isomorphic. If they are, exhibit the isomorphism, if not, explain why not.

(a)

(b)
4. How many $n$ digit decimal sequences (using the digits 0, 1, 2, \ldots, 9) are there in which the digits 1, 2, 3, 4 all appear?

5. Solve the recurrence relation

$$a_n = 3a_{n-1} + 4a_{n-2} \quad a_0 = a_1 = 1$$
6. In parts (a) and (b) determine if each graph is planar. If it is, then redraw it so it appears planar. If it is not planar, explain why.

(a)

(b)
7. In each of the following 2 graphs find an Euler circuit. If one doesn’t exist, explain why and find an Euler path; if an Euler path doesn’t exist then explain why.

(a)

(b)
8. (a) Find the generating function for $a_r$, the number of selections of $r$ balls from a pile of 5 red, 2 green and 7 blue balls.

(b) Find the exponential generating function for the number of ways to distribute $r$ (nonidentical) people into 6 rooms with between 1 and 4 (can include 1 or 4) people per room.

9. (a) How many vertices could a graph contain if it has 10 edges and all vertices of the same degree?

(b) Suppose $G$ is a connected planar graph which bounds 7 regions and has 12 edges. How many vertices does it have?
10. (a) Find a Hamilton path in this graph or prove none exists.

(b) Find a Hamilton circuit in this graph or prove none exists.
11. (a) How many ways are there to distribute 20 identical jelly beans to 5 children so that each child gets at least 1?

(b) How many ways are there to distribute 20 different toys to 5 children with each child getting exactly 4 toys?

12. Suppose you make an investment of $1000 and that the amount, \( a_n \), that this investment is worth at the end of the \( n \)th year is twice the value at the end of the previous year plus \( n \). Write out the recurrence relation for \( a_n \) and solve it.
13. Determine the chromatic number of each graph. Give a careful argument to show that fewer colors will not suffice.

(a)

(b)
14. (a) How many ways are there to arrange the letters in the word “Manhattan”?

(b) What is the probability that a random rearrangement of the letters in “Manhattan” will begin with an $M$ or $n$?

15. (a) List all nonisomorphic undirected graphs with 4 vertices.

(b) List all nonisomorphic trees with 6 vertices.