Show all work. Correct answers without supporting work will receive no credit.

(12) 1. Solve the congruence \( 57x \equiv 39 \pmod{89} \).

\[ x \equiv 3 \pmod{5} \]
\[ x \equiv -4 \pmod{13} \]
\[ x \equiv 7 \pmod{19} \]

(15) 2. Solve the simultaneous system:

3. State and prove two of the following theorems:

(A) Möbius Inversion

(B) Euler’s Theorem

(C) Wilson’s Theorem
(12) 4. Find the remainder when $3^{9647}$ is divided by 19.

(15) 5. Solve the polynomial congruence $x^2 + x + 7 \equiv 0 \pmod{27}$.

6. Prove one of the following:

(14) (A) $\sum_{n \leq x} |u(n)| \leq \frac{3x}{4}$ for $x \geq 4$.

(14) (B) If $A$ is a reduced residue system (mod $m$), $m > 2$, then $\sum_{x \in A} x \equiv 0 \pmod{m}$.

Extra Credit

(12) Prove that for any positive integer, $n$, $\phi(x) = y$ has only finitely many (perhaps none) solutions, $x$. 