1. (3 points each) Circle either “True” or “False” for each of the following:

a) **False**: Let $a$ and $b$ be non-zero whole numbers. Then $a^n \times b^n = (a \times b)^n$ for all whole numbers $n$.

b) **False**: $(9 \times 6) - (5 \times 6) = (9 - 5) \times 6$ is an example of the distributive property of multiplication over subtraction.

c) **False**: $8$ does not divide $139,738,456$.

d) **False**: $\frac{3^2}{2^2 \times 3^2 \times 13 \times 5^{10}}$ has a terminating decimal representation.

e) **False**: Every set that is closed under multiplication must also be closed under addition.

f) **False**: Let $n$ and $m$ be whole numbers with $\text{LCM}(n, m) = 15$ and $\text{GCF}(n, m) = 5$. Then $n \times m = 75$. $n \times m = \text{LCM}(n, m) \times \text{GCF}(n, m)$

g) **False**: If you were to use the prime factor test to check whether $129,411$ were prime, you would only need to check whether $129,411$ were divisible by primes no bigger than $\sqrt{129,411}$.

h) **False**: $3 \left(\frac{5}{2} + \frac{3}{4} \times 4\right) = 23$.

2. (5 points) Fill in the blanks (do not abbreviate!):

$\frac{3}{4} + \frac{3}{4} = \frac{3}{4} + \frac{3}{4}$ is an example of the **commutative** property for the addition of fractions. We can think of this property as an extension of the same property for whole number addition. We will do this by rewriting both fractions using a common denominator. We can find the lowest common denominator by looking at the **least common multiple** of six and four. Rewriting our fractions with our common denominator, we have:

$$\frac{5}{6} = \frac{10}{12}$$

$$\frac{3}{4} = \frac{9}{12}$$

We can then think of our fraction addition problem as a whole number addition problem where we are counting pieces of size $\frac{1}{12}$. We then use the corresponding property of whole number addition to get:

$$\frac{10}{12} + 9 = \frac{9}{12} + \frac{10}{12}$$

Thus, $\frac{10}{12} + \frac{9}{12} = \frac{9}{12} + \frac{10}{12}$. Rewriting in simplest form, we have $\frac{5}{6} + \frac{3}{4} = \frac{3}{4} + \frac{5}{6}$.
3. Terry’s class has 24 students. The baseball fans are fans of exactly one team. There are 4 Cub fans, 1 A’s fan, 3 Cardinals fans, and 8 Royals fans. The rest of the class isn’t interested in baseball.

(a) (2 points) What fraction of Terry’s class consists of Royals fans? Express your answer in simplest form.

\[
\frac{3}{24} = \frac{1}{8}
\]

(b) (2 points) Express your answer from (a) as a decimal.

\[
0.\bar{3}
\]

\[
3 \overline{1.\bar{3}}
\]

\[
\begin{align*}
\text{repeats!}
\end{align*}
\]

(c) (4 points) What is the percentage of Cardinals fans in Terry’s class?

\[
\frac{3}{24} = \frac{1}{8} \text{ of class are Cardinals fans}
\]

\[
8 \times 0.125 = 1
\]

\[
\text{OK}
\]

(d) (4 points) What is the ratio of baseball fans to non-fans in Terry’s class?

\[
\frac{\text{baseball fans}}{\text{non-fans}} = \frac{11}{16} \quad (4 + 1 + 3 + 8)
\]

\[
\text{non-fans} = 24 - 16 = 8
\]

\[
\frac{\text{fans}}{\text{non-fans}} = \frac{16}{8} = 2 : 1
\]

4. (4 points each) Calculate the following. If your answer is a fraction, please write it in simplest form. This is not a mental math problem! Show your work in an easy to follow manner.

a) \[
\frac{9}{16} \div \frac{3}{8} = \frac{3}{16} \times \frac{8}{3} = \frac{3}{2}
\]

\[
\frac{9}{16} \div \frac{3}{8} = \frac{9}{16} \div \frac{3}{8} = \frac{3}{2}
\]

b) \[
\frac{1}{2} \times \frac{8}{3} = \frac{1}{2}
\]
c) \[1.296 + 32.01 = \quad \begin{array}{c} 1.296 \\ + 32.01 \\ \hline 33.306 \end{array} \]

d) \[\frac{\frac{3}{5}}{\frac{1}{8}} = \frac{\frac{3}{5} \div \frac{1}{8}}{\frac{1}{8} \div \frac{1}{8}} = \frac{\frac{3}{5} \times 8}{1} = \frac{14}{5} \]

e) \[\text{GCF}(1034, 514) = \]

\[\begin{array}{c}
1034 \\
\underline{1028} \\
6 \\
\end{array} \]

\[\text{GCF}(6, 4) = 2 \]

5. (4 points) Express \(2.30\overline{7}\) as a fraction. You do not need to simplify your answer.

\[\begin{array}{c}
2.3077... \\
\underline{1000 \times 2.3077...} \\
900 \times 2.3077... \\
\hline \\
900 \\
2077 \\
\hline \\
2077 \\
900 \\
\hline \\
\end{array} \]

\[x = \frac{2077}{900} \]

6. (4 points) Write a short story problem which can be solved by \(\frac{3}{8} \div \frac{1}{12}\).

I have measured out \(\frac{3}{8}\) c brown sugar. That's \(\frac{1}{2}\) of the brown sugar on hand.

How much brown sugar is on hand? \(\frac{1}{12} \times \frac{3}{8} = \frac{3}{12} \times \frac{1}{12} \)

\[\text{Alt:} \quad \frac{3}{8} \text{ lb of chocolate} \quad \text{How many} \frac{1}{12} \text{ lb portions does the custard?} \quad \frac{3}{8} \div \frac{1}{12} \]

\[\text{Alt:} \quad \text{Tiny has travelled} \frac{1}{12} \text{ of the distance between home and school.} \]

He's gone \(\frac{3}{8}\) mile. How far is his home from school? \(\frac{1}{12} \times \frac{3}{8} = \frac{3}{12} \div \frac{1}{12} \)
7. (4 points) Explain how decomposition and regrouping are used in the following arithmetic problem:

\[
\begin{array}{c}
3.13 \\
- 3.9 \\
\hline
10.4
\end{array}
\]

In the tenths place, can’t take 9 from 3 and get a whole number. So decompose the 4 in the ones place.

\[4 = 3 + 1 = 3 + 10(\text{td})\]

and regroup those 10 tenths in the tenths place. (3 + 10 tenths = 13 tenths.) Then can subtract in each place value (each result is a whole number).

8. Consider

\[
\begin{array}{c}
2.1 \overline{63.42} \\
\hline
63 \overline{42} \\
\hline
42 \\
42 \\
\hline
0
\end{array}
\]

a) (2 points) Use estimation to show that the answer given is incorrect.

\[2.1 \times 3.2 < 3 \times 4 = 12 < 63.42 \text{ so } 3.2 \text{ is too small.}\]

At least 63.42 ÷ 2.1 > 63 ÷ 3 = 21, so one should be larger than 21. 3.2 is too small. (High number won’t help here; it overestimates result.)

b) (4 points) Show how to do the problem using the standard algorithm.

\[
\begin{array}{c}
30.2 \\
\hline
163.42 \\
\hline
63 \\
\hline
42 \\
\hline
0
\end{array}
\]

c) (3 points) Explain how to place the decimal point. In particular, why did your procedure give the correct answer?

\[
63.42 ÷ 2.1 = \frac{63.42}{2.1} = \frac{63.42 \times 10}{2.1 \times 10} = \frac{634.2}{21}
\]

Then proceed as in whole number division. (idea is to divide by a whole number.) Shift decimal point by multiplying by a power of ten. We need to multiply the dividend by the same power of ten to compensate. Thus, shift the decimal point in the dividend the same # of places in the same direction.)
9. We can factor 18 in two ways as a product of single digit numbers (up to the order of the factors); namely, 2 × 9 or 3 × 6.

a) (2 points) Using divisibility tests for single digit numbers, state a test for divisibility by 18.

\[ \frac{18}{N} \iff \frac{21}{N} \text{ and } \frac{9}{N} \]

b) (3 points) Would using the other pair of two single digit factors give a valid divisibility test for 18? Explain.

No! If \( \frac{3}{N} \) and \( \frac{6}{N} \), we've only guaranteed that \( \frac{3}{N} \) and \( \frac{(2 \times 3)}{N} \). In other words, we've only guaranteed \( \frac{21}{N} \) and \( \frac{3}{N} \). We need that \( \frac{3^2}{N} \), not \( \frac{3}{N} \) (or even \( \frac{21}{N} \)).

For example, \( 3 \mid 6 \) and \( 6 \mid 6 \), but \( 18 \nmid 6 \). Divisibility by 18 requires one factor of 2 and two factors of 3 (18 = 2 × 3²).

c) (3 points) Apply your test to check whether 18 divides 29,715,312. Show your work. Do not divide 29,715,312 by your single digit factors.

\[ 2 + 9 + 7 + 1 + 5 + 3 + 1 + 2 = 30 \]

\[ 9 \times 30, \text{ so } 9 \times 29,715,312 \]

Hence, by our test for div by 18, \( 18 \not\mid 29,715,312 \).
10. (10 points) Toby and Terry have invited you over for dinner at their new house. You have been promised your favorite beverage if you can figure out their new street address from the following clues:

(a) The address is 2 _ 9 _ Perth Avenue.
(b) The last house number on Perth Avenue is 2650.
(c) Terry’s birthday is 5/9. Both the month and the day of Terry’s birth divide the house number.

What is their street address? Is there more than one possibility? Explain.

Let \( N = \text{house#} \)

5 \| N, so by the div test for 5, the last digit of \( N \) is either 0 or 5

**Case 1** Last digit is 0

\[ N = 2n90 \]

9 \| N, so \( 9 \mid 2 + n + 9 + 0 = 11 + n \)

\[ 11 + n = 18 \implies n = 7 \]

\[ 11 + n = 27 \implies n = 16 \leq \text{too big} \]

\[ N = 2790 \]

**Case 2** Last digit is 5

\[ N = 2n95 \]

9 \| N, so \( 9 \mid 2 + n + 9 + 5 = 16 + n \)

\[ 16 + n = 18 \implies n = 2 \]

\[ 16 + n = 27 \implies n = 11 \leq \text{too big} \]

\[ N = 2295 \]

By (b), \( N \leq 2650 \), so \( N = 2790 \) is impossible.

Thus, the only possible street address satisfying the clues is 2295 Perth Ave.