The point value of each problem is given in the margin.

(10) 1. Suppose \( f(x) = 2x + 1 \) and \( x \in \{0, 1, 2, 3, 4\} \).

(a) Describe the function \( f \) as a set of ordered pairs.

(b) Write the range of \( f \) as a set.

(12) 2. If \( f(x) = 7x \) and \( g(x) = x - 5 \), with \( x \in \mathbb{R} \), find:

(a) \((g \circ f)(x)\)

(b) \((f \circ g)(x)\)

(c) \((g \circ f)(2)\)

(d) \((f \circ g)(2)\)
3. Determine if each of the following arguments is valid, and if so, give a reason for your answer.

(i) Hypothesis: If a person is named John, he is a male. The name of this person is John.

Conclusion: Therefore, this person is male.

(ii) Hypothesis: If Sam is present at a party, then he wears a red tie. If Sam wears a red tie, then he is happy.

Conclusion: If Sam is not happy, then he is not present at a party.

(iii) Hypothesis: If a person lives in Brooklyn, then she lives in New York. Joan does not live in New York.

Conclusion: Joan does not live in Brooklyn.
4. (a) Fill in the following base 2 tables:

\[
\begin{array}{c|c|c}
+ & 0 & 1 \\
\hline
0 & 0 & 1 \\
1 & 1 & 0 \\
\end{array}
\quad
\begin{array}{c|c|c}
\times & 0 & 1 \\
\hline
0 & 0 & 0 \\
1 & 0 & 1 \\
\end{array}
\]

(b) Use the tables in part (a) to compute the product \((101)_2 \times (11)_2\)

(c) Use the tables in part (a) to compute the difference \((101)_2 - (11)_2\)

(d) What law is used to justify \((3)_5(24)_5 + (3)_5(12)_5 = (3)_5(41)_5\)?
(10) 5. Circle $T$ for True and $F$ for False.

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<td>(a) If $6 \mid a$ and $2 \mid a$ then $12 \mid a$.</td>
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<td>(b) If $100013 \mid a$ and $100013 \mid b$, then $10013 \mid a - b$.</td>
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<td>(c) If $25 \mid b$ and $49 \mid b$, then $(25)(49) \mid b$.</td>
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<td>(d) If $56 \mid a$ and $56 \nmid b$, then $56 \nmid (a + b)$.</td>
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<td>(e) $(11)<em>3 = (4)</em>{10}$.</td>
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(10) 6. (a) State the Fundamental Theorem of Arithmetic.

(b) Find the prime factorization of $2100$. 
(12) 7.(i) Find all digits $n$ such that $724, n13$ is divisible by

(a) 3.

(b) 9.

(ii) Find all digits $n$ such that $57, 729, 764, n88$ is divisible by

(a) 11.

(b) 88.
8. Suppose the 435 members of the House of Representatives are placed on committees consisting of more than 2 members but less than 30 members. Each committee is to be of equal size and each member can serve on only one committee.

(a) What size committees are possible?

(b) How many committees are there of each size?

9. Suppose that we wish to compute \( a \div b \) where \( a, b \) are two integers.

(a) What restrictions must be placed on \( a \) and/or \( b \)?

(b) If \( q \) is the quotient and \( r \) the remainder in dividing \( a \) by \( b \) what is the relationship between \( a, b, q \) and \( r \)?

\[
a =
\]
(c) What inequality does the remainder satisfy?

\[ \leq r < \]

(d) Find \( q \) and \( r \) if \( a = 367 \) and \( b = 29 \).