1. Circle either “True” or “False” for each of the following (two points each):

a) **True** False: We “line up the columns” when doing the standard addition algorithm because of place value, because this is a shorthand way of expanding each addend by place value and then using the distributive property to add by place value.

b) **True** False: Range estimation can help us tell if we made an error in a calculation.

c) True **False:** Every prime number is odd. 2 is prime.

d) **True** False: If a and b are non-zero whole numbers, then GCF(a, b) ≤ LCM(a, b).

e) True **False:** $2^3 \cdot 3^5 \cdot 7$ has 3 · 5 · 1 distinct factors. (3+1) · (5+1) · (7+1) common factors.

f) True **False:** When we multiply two fractions, we must rewrite each fraction using a common denominator before we can multiply the two fractions. $\frac{5}{6} \cdot \frac{3}{5} = \frac{5 \cdot 3}{6 \cdot 5}$

g) **True** False: Suppose that a and b are non-zero whole numbers. Then dividing by $\frac{b}{a}$ is the same as multiplying by $\frac{a}{b}$.

2. Use mental math methods to evaluate the following expressions. Show enough intermediate steps so that another Math 320 student could read your solution and see how you did the calculation. (5 points each)

a) $86 + (37 + 14) = (86 + 14) + 37 = 100 + 37 = 137$

b) $298 + 63 = (298 + 2) + (63 - 2) = 300 + 61 = 361$

c) $48 \times 25 = (12 \times 4) \times 25 = 12 \times (4 \times 25) = 1200$
3. a) Evaluate \( 372 - 184 \) using the standard algorithm for subtraction. Show your work (so we can see the algorithm in action!) Do not explain the algorithm. (4 points)

\[
\begin{array}{c}
2 \ 8 \ 7 \ 2 \\
- 1 \ 8 \ 4 \\
\hline
1 \ 8 \ 8
\end{array}
\]

b) Evaluate \( 372 - 184 \) using a different algorithm for subtraction. Show your work (so we can see the algorithm in action!) Do not explain the algorithm. (4 points)

\[
\begin{array}{c}
3 \ 7 \ 2 \\
- 2 \ 8 \ 4 \\
\hline
1 \ 8 \ 8
\end{array}
\]

Other procedures include: 'adding the complement', 'see 4.2A + 12', 'subtract from the base', 'see 4.1b', 'equal additions (without the) see 4.140'

c) Explain why what you did in (a) worked for the ones place value. Telling your students that "it gave the right answer" won't help them learn and won't work here. (3 points)

**Suggestion:** Your explanation should consist of several sentences. Recall that an equation is a mathematical sentence. The word "borrow" is misleading and should be avoided.

We have 2-4 in the ones place, which doesn't give us whole numbers.

\[
372 - 184 = (3 \times 100) + (7 \times 10) + 2 - [(1 \times 100) + (8 \times 10) + 4]
= (3 \times 100) + (7 \times 10) + 2 - [(1 \times 100) + (8 \times 10) + 4]
\]

\[
= (3 \times 100) + (6 \times 10) + 0 + 2 - [(1 \times 100) + (8 \times 10) + 4]
\]

\[
= (3 \times 100) + (6 \times 10) + 12 - (1 \times 100) - (8 \times 10) - 4
\]

In the ones place, we now have \( 12 - 4 = 8 \)
4. a) Use the prime factorization method to find LCM(70, 84). Show your work. (8 points)

\[
\begin{align*}
70 & = 2 \times 5 \times 7 \\
84 & = 2^2 \times 3 \times 7 \\
\text{LCM}(70, 84) & = 2^2 \times 3 \times 5 \times 7 = 4 \times 15 \times 7 = 60 \times 7 = 420
\end{align*}
\]

b) Use the Euclidean algorithm to find GCF(39, 104). Show your work. (8 points)

\[
\begin{align*}
39 & \ | 104 & & \ | 26 & & \ | 13 & & \ | 13 \\
7 & & \ | 26 & & \ | 26 & & \ | 13 \\
1 & & \ | 13 & & \ | 13 & & \ | 13 \\
0 & & \ | 26 & & \ | 13 & & \ | 13 \\
\text{GCF}(39, 104) & = \text{GCF}(26, 39) & & \text{GCF}(13, 26) & = 13
\end{align*}
\]

5. Calculate the following. Your answer should be in simplest form. (4 points each)

a) \[ \frac{2}{3} - \frac{1}{4} = \left( \frac{2}{3} \cdot \frac{4}{4} \right) - \left( \frac{1}{4} \cdot \frac{3}{3} \right) = \frac{8}{12} - \frac{3}{12} = \frac{5}{12} = \frac{5}{12} \]

\[
\text{LCM}(3, 8) = 24
\]

b) \[ \frac{4}{9} \div \frac{2}{3} = \frac{4 \times 3}{9 \times 2} = \frac{4 \times 3}{18} = \frac{4}{6} = \frac{2}{3} \times \frac{1}{3} = \frac{2}{3} \]

\[
\frac{2}{3}
\]
c) \((\frac{3}{5} \cdot (\frac{3}{10} - \frac{1}{12})) ÷ \frac{1}{5} = \frac{9}{20}\)

\[
\frac{3}{5} - \frac{1}{12} = \left(\frac{3}{5} \cdot \frac{3}{5}\right) - \left(\frac{1}{12} \cdot \frac{1}{12}\right)
\]

\[
\text{LCM}(5, 12) = \text{LCM}(5, 12) = 2 \cdot 3 \cdot 5 = 30
\]

\[
\frac{3}{5} \cdot \left(\frac{3}{10} - \frac{1}{12}\right) = \frac{3}{5} \cdot \frac{7}{60} = \frac{21}{300} = \frac{7}{100}
\]

\[
\frac{9}{20} ÷ \frac{1}{5} = \frac{9}{20} \cdot \frac{5}{1} = \frac{45}{20} = \frac{9}{4}
\]

6. Consider the following expression:

\[4\frac{1}{2} ÷ \frac{3}{4}\]

a) Write a short story problem that would be solved by this expression. For example, "6 + 3" solves "Jeff has six apples and Sally has three apples. How many apples do they have?" (5 points)

You have prepared 4\frac{1}{2} cups of punch for a party. If you plan to serve each guest \(\frac{3}{4}\) cup of punch, how many guests can you serve?

b) Evaluate the expression. Your answer should be in simplest form. This also gives us an answer to the story problem in (a). (5 points)

\[4\frac{1}{2} ÷ \frac{3}{4} = \left(4 + \frac{1}{2}\right) ÷ \frac{3}{4} = \left(\frac{8}{2} + \frac{1}{2}\right) ÷ \frac{3}{4}
\]

\[= \frac{9}{2} ÷ \frac{3}{4} = \frac{9}{2} \times \frac{4}{3}
\]

\[= \frac{4 \times 3}{2 \times 3} = \frac{12}{6} = 2 \times 3 = 6
\]

(See answer to (a) in "6 quarts")
7. A company announced that next month each employee will receive a raise equal to \( \frac{2}{5} \) of that employee's current salary. Terry is looking forward to receiving this raise. If Terry's current monthly salary is $2400, what will Terry's salary be next month? Show your work. (7 points)

\[
\text{Salary next month} = \text{current salary} + \text{raise}
\]

\[
\text{Terry's raise} = \frac{2}{5} \times 2400 = \frac{2400}{5} = 480
\]

\[
\text{Salary next month} = 2400 + 480 = 2880
\]

\[
\text{Terry's salary next month will be} \, \$2880.
\]

Alternate solution

\[
\text{Salary next month} = \text{current salary} + \text{raise}
\]

\[
= (1 \times \text{current salary}) + \left( \frac{2}{5} \times \text{current salary} \right)
\]

\[
= \left( \frac{5}{5} \right) \times \text{current salary} + \left( \frac{2}{5} \right) \times \text{current salary}
\]

\[
= \frac{7}{5} \times 2400 = \frac{2}{5} \times 2400 = 21 \times \frac{2400}{5} = 21 \times 480 = 21 \times 120
\]

\[
= (20 \times 120) + (1 \times 120) = 2400 + 120 = 2520
\]

8. a) Using the divisibility tests we have studied, formulate a test for divisibility by 18. (5 points)

Let \( n \) be a whole number.

\[ 18 \mid n \iff 2 \mid n \text{ and } 9 \mid n. \]

Note: Although \( 18 = 3 \times 6 \), there are \( n \) such that \( 3 \mid n \) and \( 6 \nmid n \). But \( 18 \mid n. \)

b) Use your test from (a) to determine if \( 18 \mid 834 \). Show your work. This is your chance to show that you understand how to apply divisibility tests, so do not divide 834 by the numbers you gave in (a). (5 points)

Note: You will be graded on whether you correctly apply the test you stated in (a) and will not be penalized if the test you gave in (a) is incorrect.

\[ 2 \mid 834 \text{ since the one digit of } 834 \text{ is even.} \]

\[ 9 \mid 834? \quad 8+3+4 = 15, \text{ since } 9 \mid 15, \, 9 \mid 834 \]

By (a), we conclude that \( 18 \nmid n. \)

Note: \( 3 \mid 834 \text{ since } 8+3+4 = 15 \text{ and } 3 \nmid 15. \) So if we tested for divisibility

\[ 6 \nmid 834 \text{ since } 31834 \text{ but } 21834 \text{ is not.} \] So we conclude that \( 18 \mid 834 \).
c) Calculate \(834 \div 18\). Is your answer in (b) correct? (5 points)

**Note:** You're done with this problem if your answer in (b) is correct! If your answer in (b) is incorrect, please try (d) instead of redoing (a), (b), and (c).

\[
\begin{array}{c}
18 \longdiv{834} \\
\underline{18} \times 1 \quad \underline{18} \\
924 \\
\underline{924} \\
0 \\
\end{array}
\]

So \(18 \times 834\)

So the conclusion from testing for divisibility by 2 and by 9 in (b) is correct

(The conclusion from testing for divisibility by 3 and by 6 in (b) is incorrect)

d) If your answer in (b) is incorrect, fix your answer in (a) here without redoing the whole problem! Do the numbers that you’re using in your divisibility test have a common factor other than the number one? Based on this, formulate a different test for divisibility by 18. (replaces (a) = 5 points)

\[
\begin{align*}
\text{If we used} & \quad 18 \times 47 = 852 \\
\text{we get the wrong answer:} & \quad 852 \left(= 18 \times 47 \right) \\
\text{This happens because} & \quad \gcd(18, 47) = 1 \neq 1. \\
\text{Testing only 47 and 18} & \quad \text{tells that 47 and (2 \times 47) in. This guarantees that 47 has at least one factor of 2 and one factor of 3. Since } 18 = 2 \times 9 \text{ has two factors of 3, so for } 18 \text{, it must have at least two factors of 3.}
\end{align*}
\]

Now test: \(18 \times 21 = 378\)

Otherwise, yes? \(18 = 2 \times 9\)

\[\times 18 \times 1, 2 \times 9, 3 \times 6\]

So we can look at all of the possibilities for a divisibility test at the form \(a \times b + c \text{ with } a \neq b\), where \(a, b, c\) is 1, 2, 3.

**Note:** You could test all of the boxes of 18 before 1 month (some in 10 min) (some in 10 min). That's more work, but using the boxes and several other possible ideas. Also, need to be careful: Testing the divisibility by 3 using this idea would mean testing divisibility by 3 and 9. (2 times 9, 18 and 2, 3, 6) - and there are enough - for example, 20, 3, 6, and 6. 2, 3, 6. The other work for 18 becomes \(2 \times 9 + 18\), but \(2 \times 3 \times 5 \times 18 = 918\) - and 3, 9, 18 are the first 3 and 9. Divisibility tests (so you that enough factors 2 and 3)