Final Exam
December 16, 2003

Name: ________________________________

PLEASE READ THIS PAGE!!!

1. Hints:
   • You might want to quickly look over all of the questions and start by working the questions that are easiest for you.
   • Check your answers only if you have time.
   • Many of the questions have multiple parts. Don’t automatically give up on a question because you can’t do one part.
   • Consider showing your work, even when it’s not requested. This could help you earn partial credit for an incorrect answer.

2. Reminders:
   • No calculators, cell phones, PDAs, or any other electronic devices are allowed.
   • You are allowed to use the page of handwritten notes that you prepared for use with this exam. No other type of written materials is allowed.

3. Read the following statement and sign your name:
   I affirm that I neither will give nor receive unauthorized assistance in this examination. All work that appears on the following pages is entirely my own.
   Signature: ________________________________________

4. Please make sure that your exam contains eleven pages, including this one.

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Total: 200
1. Circle either “True” or “False” for each of the following: (three points each)
   a) True False: A statement about all whole numbers is true if it is true for all of the examples that you have checked.
   b) True False: Let $a, b$, and $c$ be whole numbers. Suppose that $a \neq 0$. Then it is always true that $a^b \cdot a^c = a^{b+c}$
   c) True False: Let $a, b$, and $c$ be whole numbers. $a + (b + c) = (a + b) + c$ is an example of the associative property.
   d) True False: Each side of an equilateral triangle is 3 inches long. The perimeter of this triangle is 9 inches.
   e) True False: Suppose that you are counting in base six. You have just gotten to 555$_6$. The next number will be 560$_6$.
   f) True False: Let $n$ and $m$ be non-zero whole numbers. Suppose that $n > m$. Then $\frac{1}{n} < \frac{1}{m}$.
   g) True False: The letter “A” has reflection symmetry, but not rotation symmetry.
   h) True False: For all circles of radius $r > 0$, the ratio of circumference to diameter is equal to $\pi$.

2. You ask your sixth grade students about their hobbies. Five of your students take music lessons. Ten of your students play soccer. Three of your students take music lessons and also play soccer. There are 28 students in your class.
   a) How many of your students play soccer and do not take music lessons? (four points)
      *Hint:* You might want to draw a Venn diagram.
   b) If we let $A = \{x : x$ is a student in your class$, B = \{x : x$ takes music lessons$\}, and $C = \{x : x$ plays soccer$\}$, describe a typical element of the set $A - (B \cup C)$. (four points)
3. Please convert the following numerals to base 10. Show your work. (four points each)
   a) $500_8 =$
   
   b) MCMLXIV =$
   
   Recall that in the Roman numeration system, M = 1000, D = 500, C = 100, L = 50, 
   X = 10, V = 5, and I = 1.

4. Find the missing term in each of the sequences below. Also circle the sequence type. (two points 
   for the missing term, two points for the sequence type.)
   a) 2.2, 3.4, 4.6, 5.8, _______   Arithmetic   Geometric   Neither
   
   b) 1, 0.5, 0.25, 0.125, _______   Arithmetic   Geometric   Neither
   
   c) $\pi$, $\pi^2$, $\pi^3$, $\pi^4$, _______   Arithmetic   Geometric   Neither
5. Terri just learned about the division algorithm for whole numbers. Her little brother Tom is helping her practice. After Tom tells Terri a whole number, she tells him the remainder she gets after dividing that number by 3.

   a) List all of Terri’s possible answers, provided that she divides correctly. Explain. (four points)

   b) Does this define a function on the whole numbers? Explain. (four points)

   c) Terri is starting to get bored. So, after Tom tells her a whole number, she looks at her watch and tells him the last digit of the current minute. Does this define a function on the whole numbers? Explain. (four points)

6. Please find the following. Use any method you’d like and show your work. (five points each)

   a) \( LCM(24, 56) \)
7. Use divisibility tests to determine whether 12345060 is divisible by 6. Show your work. (ten points)

8. Fran is learning how to divide decimals. Here’s the problem that she’s working on: 2.625 ÷ 1.25. Fran is getting an answer of 0.21.
   a) Is Fran’s answer correct? Why or why not? (five points)
   b) Explain where to put the decimal place when you’re dividing decimals. Use Fran’s problem as an example. (five points)
9. a) Calculate \(4\frac{3}{8} \div \frac{5}{2}\) using fractions. Give your answer in simplest form. (four points)

b) Convert \(4\frac{3}{8}\) and \(\frac{5}{2}\) to decimals. (four points)

c) Calculate \(4\frac{3}{8} \div \frac{5}{2}\) using the decimals that you found in (b). (four points)

10. Which of the following numbers have a terminating decimal representation? (four points each)
   a) \(\frac{15}{300}\)

   b) \(\frac{2^3 \cdot 3^{11}}{2^8 \cdot 3^4 \cdot 5^2}\)
11. Tim sees an ad in the paper from a local car dealership that ends with “We’ll reduce the price of this luxurious, sleek, high-performance $50,000 sedan by 20% on the first day of our sale. And, each day, we’ll reduce its price by 5% of the sale price until this fantastic vehicle is sold!!!”

a) What is the car’s price on the first day of the sale? (five points)

b) Write an expression for the car’s price on the third day of the sale. Do not calculate the car’s price. (five points)

c) If the car is still available on the seventeenth day of the sale, will it be free? Explain. Note that we aren’t concerned with sales tax, dealer’s fees, and the like here. (five points)

12. The map of Manhattan, KS in the SBC Yellow Pages is drawn with a scale of \( \frac{15}{16} \) inches equals \( \frac{1}{2} \) mile. Your car is in the shop, so you’re thinking about walking to school. After carefully tracing your route to school on this map, you figure that your apartment is \( 2 \frac{3}{16} \) inches on the map from Hale Library. What is the distance in miles between your apartment and Hale Library? Write your answer as a mixed fraction in simplest form. Don’t forget to include the units! (ten points)
13. Convert $2.\overline{183}$ to a fraction. Do not convert your answer to simplest form. (five points)

14. a) Draw an example of a convex polygon. (four points)

b) Draw an example of a polygon that is not convex (four points)

c) Draw an example of two parallel lines with a transversal. Indicate a pair of corresponding angles. Should these two angles be the same size? (four points)
15. Find all of the missing angles in the figure below and fill in the list below. Show your work. (ten points)

A: 
B: 120°
C: 
D: 

16. Find the shaded area in the figure below. Show your work. Don’t forget to include the units in your answer! (12 points)
17. Consider a right hexagonal prism which has a height of \( \frac{1}{3} \) yard. The length of each side of our regular hexagon (the base of our prism) is 120 inches. It is okay to leave square roots of a whole number in your answers to this problem. Don’t forget to include the units in your answers!
Recall that 12 inches = 1 foot and that 3 feet = 1 yard.
(Hint: A regular hexagon can be divided into six equilateral triangles if you draw lines through the center of the hexagon).

a) Find the surface area in square feet of our prism. (six points)

b) Find the volume in cubic feet of our prism. (six points)
18. Choose one of the following questions. Please include the appropriate letter (a, b, or c) with your answer. (ten points)

a) Draw a picture of two lines intersecting and label the angles. Choose a pair of vertical angles and show that the two angles have the same measure.

b) State the formula for the surface area of a right circular cylinder with a base of radius \( r \) units and a height of \( h \) units. Show why this formula is true. Draw a picture as part of your explanation.

c) Suppose that we have a rectangular prism which has a square as its base. Each side of the square base has length \( a \) units and the prism has height \( h \) units. If you also have a cylinder of height \( h \) units, what should its radius be to guarantee that its volume is the same as the volume of our rectangular prism? Explain.