Show all your work in the space provided under each question. Please write neatly and present your answers in an organized way. You may use calculators, but books and notes are not allowed.

(50) 1. a rancher needs to provide a daily dietary supplement to the camels on his ranch. He needs 6 units of protein and 5 units of carbohydrate to add to their regular feed each day. Feed-Ex costs 32¢ per pound and supplies 4 units of protein and 8 units of carbohydrate per pound. Feed-Why costs 12¢ per pound and supplies 3 units of protein and 2 units of carbohydrate per pound.

The linear programming problem is to determine the number of pounds of the two brands to be purchased, in order to fulfill the camels’ dietary needs at the least cost.

(a) Write down the linear constraints, and the objective function to be minimized, for this problem.
(b) Set up the initial tableau for this problem.

(c) Perform a first pivot-step for the tableau in part (b), consistent with the simplex algorithm. (I.e., choose a correct entry to pivot with, and then go ahead and pivot.)

(d) Write the problem in matrix form.

(e) Write the dual of the problem in matrix form.
(30) 2. Consider the problem: \( \text{Minimize } 10x + 12y \text{ subject to:} \)

\[
\begin{align*}
  x + 2y & \geq 1 \\
  -x + y & \geq 2 \\
  2x + 3y & \geq 1 \\
  x & \geq 0, \quad y \geq 0.
\end{align*}
\]

(a) State the dual problem.

(b) Solve the dual problem, by the simplex method. (Only one pivoting step is necessary, as it happens.)

(c) From your solution in (b), read off the values of \( x \) and \( y \) which solve the original problem.
(20) 3. Our famous furniture-manufacturing problem has the initial tableau:

\[
\begin{bmatrix}
0 & 3 & 1 & 0 & 0 & 0 & | & 96 \\
1 & 1 & 0 & 1 & 0 & 0 & | & 18 \\
2 & 6 & 0 & 0 & 1 & 0 & | & 72 \\
-80 & -70 & 0 & 0 & 0 & 1 & | & 0
\end{bmatrix}
\]

and the final tableau:

\[
\begin{bmatrix}
1 & 0 & 1/3 & -1 & 0 & 0 & | & 14 \\
0 & 1 & -1/3 & 2 & 0 & 0 & | & 4 \\
0 & 0 & 10/3 & 60 & 0 & 1 & | & 1400
\end{bmatrix}
\]

Recall that the figures 96, 18, 72 are the labor-hours available for carpentry, finishing, and upholstering, and that the problem is to determine how many chairs and how many sofas to make in order to maximize the profit (which is $80/chair and $70/sofa). The final tableau gives the answer of 14 chairs and 4 sofas, for a total profit of $1,400.

(a) Suppose the number of labor-hours available for finishing decreases from 18 to 16. How many chairs and sofas should be made in that case? And what will be the total profit?

(b) For what range of values of \( h \) will a marginal analysis on the effect of a change of \( h \) labor-hours for finishing be valid?