(20) 1. Find all of the solutions to the following systems of linear equations, using the method of gaussian elimination.

\[
\begin{align*}
x - 3y + 2z &= 10 \\
-x + 3y - z &= -6 \\
-x + 3y + 2z &= 6
\end{align*}
\]

In detail –

(a) Produce the row-reduced matrix that arises from this system, by following the gaussian elimination procedure.
(b) Interpret the result of part (a), so that all the solutions of the system are expressed.

(18) 2.(a) Write the system in problem 1 in the form of a matrix equation $AX = B$.

(b) Is the matrix $A$ invertible? Explain your answer briefly.
3. (a) Find the inverse of the matrix \( A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix} \).

(b) Use the result of part (a) to solve the matrix equation

\[
\begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix} \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}
\]
4. Our corporation has two factories (factory A and factory B), and each factory manufactures two items (item I and item II). Each day, factory A produces 600 of each item I at a total cost of $15,000, factory B produces 1000 of item I and 600 of item II, at a total cost of $18,000. An order is received for 30,000 of item I and 24,000 of item II.

Problem: for how many days should each factory operate to fill the order at the least cost?

(a) Determine the linear constraints for this problem.

(b) Sketch the feasible set for this problem on the facing page (back of page ?).

(c) Using whatever method you like (we've covered two in class), find the answer to the problem.
A feasible set for a certain linear programming problem is given, roughly, above, with the line segments labelled by their slopes.

(a) At which corner(s) (A through E) of the feasible set is the objective function $2x + 3y$ maximized?

(b) At which corner is the same objective function minimized?