1. The vectors below form a spanning set for a subspace of $M(3,1)$. Find a basis for that subspace. What is its dimension?

\[
\begin{bmatrix}
2 \\
1 \\
3
\end{bmatrix}
\quad \begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix}
\quad \begin{bmatrix}
5 \\
3 \\
8
\end{bmatrix}
\]

2. Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects points in the $y$ axis. Find the matrix which describes this transformation.
3. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Find $A^{-1}$.

4. Define $T: \mathbb{R}^3 \to \mathbb{R}^5$ by $T(x) = Ax$ where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Find a basis for the image of $T$. Find dim (image $T$).
5. Suppose $T : V \to W$ is a linear transformation of the vector space $V$ to the vector space $W$. Suppose $\{V_1, V_2, V_3\}$ is a dependent set of vectors in $V$. Show that the set $\{T(V_1), T(V_2), T(V_3)\}$ is dependent in $W$.

6. Find a transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ which rotates $\mathbb{R}^3$ by $\frac{\pi}{4}$ radians around the $z$-axis. (When viewed from the positive $z$-axis.)
7. Define $T : \mathbb{R}^4 \to \mathbb{R}^3$ by $T(v) = Av$ where

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{bmatrix}.$$  

Find a basis for the image of $T$ and a basis for the nullspace of $T$.

8. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with the property that $T(x) = b$ always has a solution for every $b \in \mathbb{R}^m$. What can you say about the relative sizes of $m$ and $n$? Explain your answer.
9. Find the $LU$ decomposition of the matrix

\[ A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & 2 & 6 \end{bmatrix}. \]

10. Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ whose image is the line $y = 4x$. 