Show all your work in the space provided under each question. Each problem is worth 10 points.

1. Find all solutions to the system of equations. (Show your steps.)

   \[ x + y - 3z = -1 \]
   \[ 3x - y + z = 7 \]
   \[ 5x + 3y - 5z = z \]
2. Let \( x = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \) and \( y = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \) in \( \mathbb{R}^3 \). Is there any element in their span with all positive entries? Prove your answer.
3. Let $A = (2, 0)$. Give a geometric description of all vectors $B$ in $\mathbb{R}^2$ such that $B \cdot A \geq |B|$.
4. Are the matrices \[
\begin{bmatrix}
2 & 0 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
2 & 3 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
3 & 0 \\
2 & -1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\] linearly independent in \(M(2, 2)\)?
5. Find a spanning set for the nullspace of the matrix \[
\begin{bmatrix}
1 & 1 & 2 \\
3 & 2 & 5 \\
2 & 1 & 3
\end{bmatrix}.
\]
6. Let $S = \{A, B, C, D\}$ be a set of $m \times n$ matrices. Suppose that $D = 2A + B + C$ and $C = B - A$.

(a) Is the set $\{A, B, D\}$ independent? Explain.

(b) Is the set $\{A, C, D\}$ independent? Explain.
7. Is the matrix \[
\begin{pmatrix}
1 \\
-1 \\
2
\end{pmatrix}
\] in the span of the matrices
\[
\begin{pmatrix}
1 \\
0 \\
1
\end{pmatrix}
\begin{pmatrix}
1
\end{pmatrix}
\begin{pmatrix}
1 \\
1 \\
-2
\end{pmatrix}
\begin{pmatrix}
-2
\end{pmatrix}
\begin{pmatrix}
-1
\end{pmatrix}
\begin{pmatrix}
-2
\end{pmatrix}
\]
Prove your answer.
Let $x$, $y$, $z$ denote the number of cars passing the points labeled $x$, $y$, $z$ on the diagram. Write a system of equations which describes this network. Solve this expressing your answer as $z = s$, with expressions for $x$ and $y$ in terms of $s$. If $z = s$ what is $x$ and $y$?
9. Draw a picture of the set $\{sv + tu : -1 \leq s \leq 1, -1 \leq t \leq 1\}$ where $v = (1, 0)$ and $u = (1, 1)$ in $\mathbb{R}^2$. Label vertices and intercepts of the figure you find.
10. True or false. No justification is required.

_____ A) In \( \mathbb{R}^2 \) the span of \((1, 2)\) is a line.

_____ B) If \( S \) and \( T \) are subspaces of a vector space \( V \) then so is \( S \cap T \).

_____ C) If \( S \) and \( T \) are subspaces of a vector space \( V \) then so is \( S \cup T \).

_____ D) If \( S \) and \( T \) are subspaces of a vector space \( V \) then so is \( S + T \).

_____ E) In \( \mathbb{R}^3 \), \[
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\]
where \( a \neq 0 \), \( b \neq 0 \), \( c \neq 0 \) are linearly dependent.

_____ F) The nullspace of an \( m \times n \) matrix is a subspace of \( \mathbb{R}^n \).

_____ G) The column space is the same as the nullspace of a matrix.

_____ H) A homogeneous system \( Ax = 0 \) is always consistent.

_____ I) \( Ax = b \) is solvable if and only if \( b \) is in the column space of \( A \).

_____ J) The nullspace of an \( m \times n \) matrix is a subspace of \( \mathbb{R}^m \).