Name: ____________________________

Recitation: ________________________

**Math 240**  
**Exam 1**  
**Feb. 14, 2006**

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**Total**

Closed book. You may use a calculator and one 8 ½ x 11" sheet of handwritten notes (both sides). You must show your work to receive full credit. Write solutions in explicit form if possible. Each problem is worth 10 points.

**Pledge:**  
On my honor, as a student, I have neither given nor received unauthorized aid on this examination: ____________________________

(signature)  
(date)
1. Find all solutions to $\frac{dy}{dx} = 3x + 2y$. 
2. Solve the initial value problem \( \frac{dy}{dx} = \frac{x^2 + y^2}{y^2 - 2xy}, \quad y(0) = 1. \)
3. Find $y(1)$ where $y(x)$ is the solution to the initial value problem,
\[ \frac{dy}{dx} = y^2 - y, \quad y(0) = 1.5. \]
4. Find all solutions to \( \frac{dy}{dx} = \frac{2xy}{x^2 + y^2} \).
5. Using the improved Euler method with step size $h = 0.5$, approximate $y(1)$ if \( \frac{dy}{dx} = 2x - y^2, \quad y(0) = 2. \)
6. Find and classify (as stable, unstable, or semi-stable) the equilibrium points of \( \frac{dP}{dt} = P^2 (4 - P^2)(25 - P^2) \).
7. Show that $2 < y(2) < 3$ for the solution to the initial value problem,
\[
\frac{dy}{dx} = 1 - x^2 + 2xy - y^2, \quad y(0) = 1.
\] One way to do this is to show that $x < y(x) < x + 1$ for $x > 0$, but there are other ways that work as well.
8. In learning theory, an autonomous equation of the form \[
\frac{dC}{dt} = rC(a - \ln(C))
\] is sometimes used to model the percentage of correct answers \(C\) as a function of instructional time \(t\), where \(r\) and \(a\) are constants (this is called a Gompertz equation). If \(\lim_{t \to \infty} C(t) = 0.85\), what, if anything, can you conclude about the parameters \(r\) and/or \(a\)?