(15) 1. A mass distribution occupies the region in the first octant which is enclosed by the surfaces $y = x^2$, $y = x$, $z = 1$, $z = 2 + x$. If the mass density function is $\delta(x, y, z) = 10y$ units of mass/unit volume, calculate the total mass in the region.
(15) 2. A mass distribution occupies the region which is above the surface \( z = \sqrt{x^2 + y^2} \) and under the plane \( z = 2 \). If the mass density function is \( \delta(x, y, z) = z^2 \), use a triple integral in spherical coordinates to calculate the total mass.
3. Calculate the area of that part of the paraboloid $z = x^2 + y^2$ which is between the planes $z = 1$ and $z = 9$. 
(15) 4. The force field \( \vec{F} = y\vec{i} - x\vec{j} \) acts on an object as it moves in the plane. Calculate the work done by \( \vec{F} \) as the object moves from \((0,0)\) to \((3,1)\) by going from \((0,0)\) to \((1,1)\) along the curve \( y = x^2 \) and then along the line segment from \((1,1)\) to \((3,1)\).
(10) 5. Show that the force field

\[ \vec{F} = \left( y - \frac{1}{x^2} \right) \hat{i} + \left( x - \frac{1}{y^2} \right) \hat{j} \]

is conservative in the region \( x > 0 \) and \( y > 0 \) by finding a potential function for \( \vec{F} \). Now use the potential function to find the work done by \( \vec{F} \) as it acts on an object which moves along a curve from \((1,1)\) to \((3,2)\) in the region \( x > 0 \) and \( y > 0 \).
(15) 6. Use Green’s theorem to evaluate the line integral

\[ \int_C (1 + xy) \, dx + (x^2 + y) \, dy \]

where \( C \) is the triangle with vertices \((0,0), (2,0), (0,2)\) directed counterclockwise.
7. Evaluate the surface integral \( \int \int_S \sqrt{x^2 + y^2} \, dS \) where \( S \) is the parametrized surface \( x = s \cos t, y = s \sin t, z = t \) for \( 0 \leq s \leq 1, 0 \leq t \leq 2\pi \).