CALCULUS III

NAME _______________________

EXAM II
Rec. Instr. _______________________

SPRING 1998
Rec. Time _______________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. An object is moving in 3-space according to the parametric equations $x = t^2$, $y = \sin t$, $z = \cos t$. Find $a_T$, $a_N$ and the curvature $\kappa$ as functions of the time $t$. 
2. An object is moving in the plane along the curve $y = \frac{1}{2}x^2$. It is moving from left to right at a constant speed of 4 ft/sec.

a) Find $a_T$ and $a_N$ when the object is at the point \( \left( x, \frac{1}{2}x^2 \right) \).

b) Find the velocity vector and the acceleration vector when the object is at the point \( \left( 1, \frac{1}{2} \right) \).
3. An object is moving in the plane. At a certain instant, say \( t = 2 \) seconds, you know that \( \vec{r}(2) = \vec{i} - 2\vec{j} \), \( \vec{v}(2) = 2\vec{i} + \vec{j} \) and \( \vec{a}(2) = -\vec{i} + \vec{j} \)
for the position vector, velocity vector and acceleration vector respectively. Answer the following questions. Do not attempt to find \( \vec{r} \), \( \vec{v} \) and \( \vec{a} \) as functions of time.

At \( t = 2 \) seconds,

a) where is the object located?

b) what is the speed of the object?

c) what is the unit tangent vector \( \vec{T} \)?

d) what is \( a_T \)?

e) what is \( a_N \)?

f) Is the object speeding up or slowing down? Why?
4. Let \( f(x, y) = 3x^2y + 2x^3 + 3y^2 + 1 \).

a) Find the equation of the plane tangent to \( z = f(x, y) \) at the point \((1, -2, 9)\).

b) Find all points on the surface \( z = f(x, y) \) at which the tangent plane is horizontal.
(15) 5. A quantity $Q$ depends upon $x$ and $y$ according to $Q = xe^{x^2y}$. Both $x$ and $y$ are changing with the time $t$ and at a certain instant you know that $x = 2$, $y = \frac{1}{2}$, $\frac{dx}{dt} = 2$ and $\frac{dy}{dt} = -2$. Use the chain rule to find $\frac{dQ}{dt}$ at this instant.
6. Suppose that \( z = f(x, y) \) and at the point \((x, y) = (2, 3)\) you know that \( \frac{\partial f}{\partial x}(2, 3) = 4 \) and \( \frac{\partial f}{\partial y}(2, 3) = 2 \). If \((r, \theta)\) denote polar coordinates in the \(xy\) plane, use the chain rule to calculate \( \frac{\partial z}{\partial \theta} \) and \( \frac{\partial z}{\partial r} \) when \( x = 2 \), \( y = 3 \).
(5) 7. Determine whether \( f(x, y) = \frac{1}{2} \ln(x^2 + y^2) \) satisfies the partial differential equation \( \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \), by actually calculating the partial derivatives involved.